MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXII. APRIL 12, 1912. No. 6

F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Francis Gilbert Brown, 68 Murillo Road, Lee, S.E.;
The Rev. Thomas Fenton Fyffe, B.A., Trindon Grange, R.S.O., Co. Durham;
The Rev. Charles Whitacre, 28 Earl Street, Stafford; and
John Woodrow, Coats Observatory, Paisley,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

The Rev. John Turnbull Bird, M.A., Chaplain to the Forces, care of Sir C. R. M'Grigor, Bart., & Co., 25 Charles Street, St. James's Square, London, S.W. (proposed by Col. E. E. Markwick);

Charles Frederick William Halliley, Badulla, Ceylon, and Junior Conservative Club, 43 Albemarle Street, London, W. (proposed by F. Hugh Capron);

Ernest W. Hardy, F.R.G.S., Head Master, Westing Schoolhouse, Uyea Sound, Shetland, N.B. (proposed by R. Falcon-Cooke); and

Friedrich Wilhelm Ristenpart, Ph.D., Director of the Observatory, Santiago de Chile (proposed by F. W. Dyson).

Professor Hermann Albert Kobold, Director of the Centralstelle, Kiel, Germany,

was proposed by the Council as an Associate of the Society.

Seventy-two presents were announced as having been received since the last meeting, including, amongst others:—

T. W. Backhouse, Catalogue of 9842 Stars, or all Stars very conspicuous to the naked eye for the epoch of 1900; and E. W. Maunder, the Science of the Stars, presented by the authors.

Harvard Observatory Annals, vols. lvi., No. 4, and lxi., part 3 (Mrs. Fleming, stars having peculiar spectra, and W. H. Pickering, Statistical investigation of Cometary Orbits), presented by the Observatory.

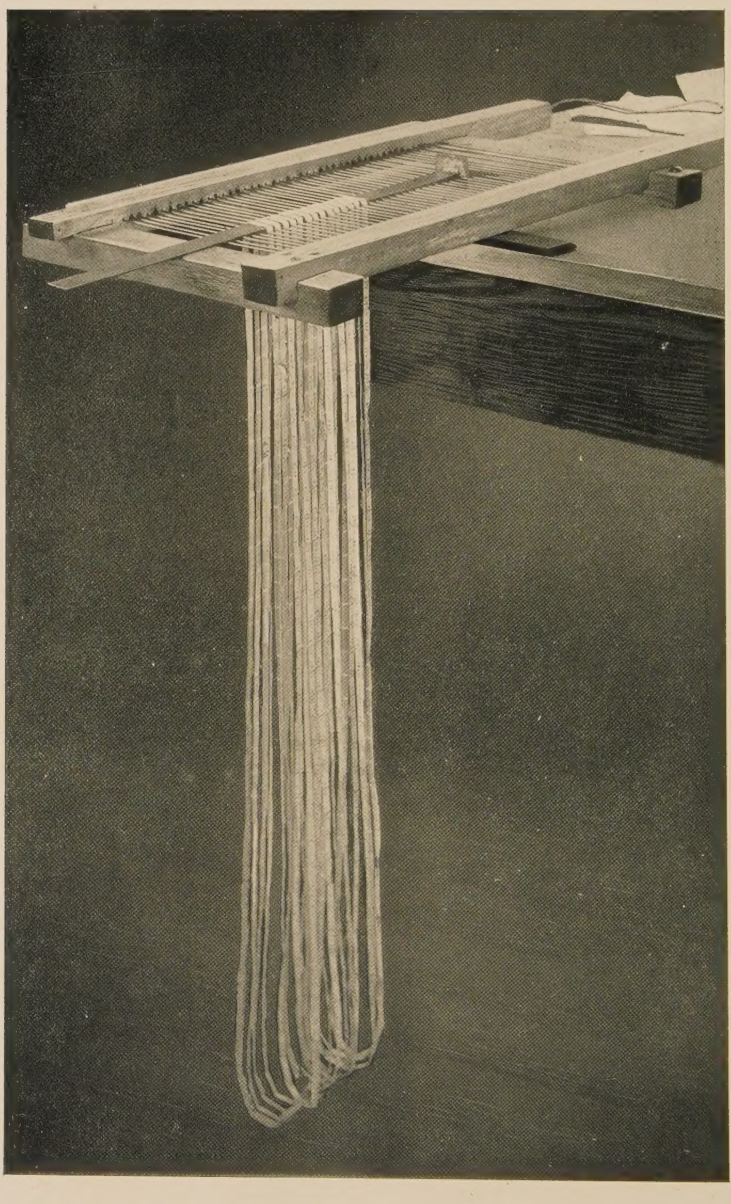
On a Device for Facilitating Harmonic Analysis and Synthesis. (Plate 5.) By Ernest W. Brown, M.A., Sc.D., F.R.S.

1. It has been mentioned in previous papers that some mechanical device was much needed for dealing with the large number of small harmonic terms which are present in the expressions for the moon's place. Several plans for doing this work mechanically have been drawn up and rejected, chiefly because they required the attendance of either a skilled artisan or a skilled computer. The problem, briefly stated, consists of two parts. One is the arrangement of the large masses of numbers which have to be added in different ways; the other is the actual addition of these numbers.

2. The introduction of the comptometer has made the second part of the problem quite simple. No matter how long the column of figures, the addition can be done rapidly and accurately without concentrated attention. Unless it be desired to print every number as it is added to the total, those forms of adding machines in which the simple pressure on a key performs both the addition of the digit and the "carrying over" are more economical of time than those machines in which a lever must be pulled to do this work after the proper keys have been depressed. The point is not of importance when each number consists of many digits, but it makes a considerable difference when there are many numbers of from one to three digits each. The chief saving of time in the use of a machine, however, is gained by an operator who can strike the correct keys with his right hand without looking at the machine, so that his attention is not distracted from the column of figures he is following with his left hand. In order to do this easily, he is frequently recommended not to use the keys 6, 7, 8, 9, but to add those digits by

Digitized by the Internet Archive
in 2026

https://archive.org/details/monthly-notices-of-the-royal-astronomical-society_1912-04-12_72_6



A DEVICE FOR FACILITATING HARMONIC ANALYSIS AND SYNTHESIS.
ERNEST W. BROWN.

striking 3 + 3, 3 + 4, 4 + 4, 4 + 5, respectively. Further assistance is afforded by the construction of the faces of the keys; the even numbers have flat faces and the odd hollow faces, and the sense of touch is of great assistance in avoiding errors. All the skill needed is learnt in three or four weeks, so that the expense of operation is small.

3. The difficult part of the problem is the arrangement of the numbers to be added so that the operator may not need constant direction and is not required to spend a considerable part of his time over it. The device described here has been made for this purpose and is now in use. It consists of tapes on which the numbers to be added are written, a carrier which holds these tapes, and a frame on which the carrier is placed, with guides to prevent the tapes from becoming entangled or getting out of position. The dimensions of the various parts can be varied at will. Those adopted and described here were considered to be best suited to the particular work in hand. No part requires any special accuracy of construction.

The "tapes" consist of rectangles of cardboard $\frac{3}{16}$ in. by $\frac{3}{4}$ in. pasted lengthwise on a heavy cambric tape, $\frac{1}{16}$ in. being left between the ends of adjacent cards. The width of the tapes is $\frac{3}{16}$ in., and various lengths are necessary according to the work in hand.

The frame is rectangular, and consists of four pieces of oak of cross section 1 in. by 1 in., two of them being 24 ins. long and the other two 12 ins. The illustration (Plate 5) shows the method of joining them together, the shorter pieces being let into the longer ones, which are 7·5 in. apart, to a depth of $\frac{1}{2}$ in., and fastened in position by screws.*

Before the frame was put together the middle of each side was bored with 47 (an arbitrary number) holes $\frac{5}{16}$ in. apart, the holes on one side being made to a depth of $\frac{1}{2}$ in. and on the other side right through. After the frame had been screwed together, the guides (ordinary knitting needles 8 in. long) were inserted through the open holes and pushed through into the opposite holes. They are prevented from falling out by a piece of wood $\frac{1}{2}$ in. by $\frac{1}{4}$ in. cross section, extending the whole length of the outer side of the frame so as to cover all the outer ends of the open holes, and fastened with screws. The holes were so drilled that the guides are not tightly held. The frame is screwed on to a table with one end near the edge in order that the portion carrying the guides may project over the edge.

The carrier is a flat piece of brass $\frac{1}{16}$ in. by $\frac{3}{4}$ in. cross section and about 18 in. long. On one end, perpendicular to its length, a piece of brass about 1·5 in. square and of small thickness was soldered, so that when the carrier is laid on the frame it will not slip across the guides.

* The frame at present in use was made by the writer. The tapes were made to order by the firm of Bradley & Scoville of New Haven, and cost from one to two dollars per length of 1000 cards.

The illustration shows the device arranged for the formation of a double-entry table (see No. 13). The method of using it is shown in the various applications described below.

The Summation of Harmonic Terms at Equal Intervals of Time.

4. In the summation of harmonic terms at numerous dates we can always limit the number of values of any one term which need be calculated. This number is such that the maximum difference between two consecutive values shall be less than the pre-assigned error which is to be allowed. The intervals at which the term is to be computed are not usually in an exact ratio to the period of the term. Suppose that the interval between the successive values required is taken to be unity, and let there be a of these intervals in the period of the term. The period a is not a whole or fractional number in general; let it be expressed by a convergent p/q. If we divide the argument into p parts and let it run through q periods, every value corresponding to each part will occur once and only once. Hence the convergent must be so chosen that this number, p, of parts shall give all the values of the function to the required degree of accuracy. In the choice of the most suitable convergent, the value of q should be as small as is consistent with this condition. The unit interval consists of q of the p parts into which the argument is divided. Since p and q are prime to one another, if we start at any one of the p values and take the values successively at unit intervals, that is, at every q parts of the argument, all the p values will occur and we shall run through q periods of the term before returning to the initial value.

5. Use is made of these facts by taking a length of the tape containing p cards and joining the ends. The p values of the function are written on successive cards of the tape in the order in which they occur when unit intervals are taken. Suppose that the convergent is 47/15. After computing the 47 values, we write them on the cards of the tape in the order 0, 15, 30, 45, 13 (=60–47), 28, 43, 11 (=58–47), 26, . . . Thus, wherever we start on the tape the following card always gives the value of the term one unit interval later. Every term to be included is treated in this way. The unit interval is the same for all, but the periods and convergents will differ.

After each term has been entered on its tape the tapes are placed over the carrier, the hanging portions dropped between the guides, and the carrier is then lowered until it rests on the frame. One card of each tape then rests on the face of the carrier.

The argument of each term for the first date required is computed. As there will generally be more than one complete period on each tape, that number amongst the arguments on the tape must be chosen which gives a value of the function nearest to the value at the given date. The tape is then moved over

the carrier (which is meanwhile held fixed) until the proper argument appears on its face.

After all the tapes have been thus adjusted the numbers are ready for summation. The sum of those appearing on the face of the carrier is the sum of the harmonic terms at the first date. This is found on the comptometer and recorded. The carrier is then turned through 180°. This turn brings the next succeeding card of each tape on to the face of the carrier. According to the process of formation described above, the numbers so appearing are the values of the terms a unit interval later. They are summed and recorded. The process is continued until a certain number of sums has been obtained.

6. Since the period used is only an approximation to the actual period, there will be an error which will gradually accumulate in each term, so that after a number of turns of the carrier the cards on its face will no longer correspond to the date which has been reached. With a given number of turns the amount of the error in any term will depend on the convergent adopted, and will be less the closer the convergent is to the actual period. Hence the number q of periods on a tape should be as great as is possible without making the tapes of inconvenient length. At a date when the error becomes sensible, the tapes must be readjusted. This requires that the arguments for this date be computed with the exact period. Suppose that 50 consecutive sums can always be obtained with the required accuracy by simply turning the carrier. The operator must be given the new arguments to be used for each tape after every 50 intervals. For 5000 sums he will need 100 arguments for each term. These are all given to him at the start of the work, so that he may do the whole of it without supervision. The formation of these arguments is not difficult and can be done mechanically on an arithmometer.

The readjustment to the new arguments has been found to require but little time or trouble. The results can be tested by differences and by duplicate calculation. If the latter is done backwards with the same set of arguments, most of any accumulated error will disappear on taking the mean between the two values for the same date obtained in the two calculations.

7. Various modifications to deal with special cases may be adopted. The frame is wide enough to admit of at least four carriers side by side. When some of the periods are very long, the corresponding tapes would be of an inconvenient length. We can take an integral number l of time-intervals as our unit for these. The convergents to periods consisting of a time-intervals are then found for the number a/l instead of for a. The carrier with these long-period terms must be turned once to every l turns of the other carrier. The signal for turning may be given by an extra tape on the latter carrier.

Sometimes the convergents are inconvenient, especially when one gives too small a value of p and the next so large a value

that the tape will be inconveniently long. This difficulty has been overcome in some cases by a device suggested by Mr. Hedrick, who has also greatly assisted in working out all the details in the methods of this paper. The term is computed with the smaller convergent but with half the given amplitude, and the numbers are written on a tape in the usual manner. The same series is copied on another tape. If we add the numbers on the two tapes having the same argument we get the value of the term for one of the p parts; if we displace one of the tapes so that the p^{th} part on one is added to the $(p+1)^{\text{th}}$ part on the other we obtain a value of the term which is half way between these parts.* Of course three tapes may be used in this way if necessary, so as to secure a division of the circumference into $3p$ parts.

In one case two successive convergents 12/1, 743/62 were found. The convergent 12/1 will permit of a run for 50 successive sums, but the amplitude was such that p should be about 120. The difficulty has been solved by using two tapes with periods having convergents 119/10, 121/10, respectively, and with amplitudes equal to half the amplitude of the term. These can always be so adjusted that the sum of the two values appearing on the face of the carrier is the correct value of the term for a run of 50 sums, within the limits of error allowed.

8. If we need the sum of the sines as well as that of the cosines, two carriers may be placed side by side on the frame, each tape passing over both of them. A tape is drawn up and a loop of it dropped between the carriers. This loop is adjusted to correspond to one quarter of the period of each term. Each tape is treated in the same way. The numbers on one carrier when added give the sum of the cosines; those on the other, the sum of the sines. To get these sums for the next date each carrier is turned through 180°.

The Formation of a Double-Entry Table.

9. Suppose that it be required to form a double-entry table of the terms †

$$S = \Sigma_{i,j} A_{i,j} \{1 + \cos(i\theta + j\phi + \alpha_{i,j})\},$$

where i, j are integers, θ, ϕ variable arguments, and A, α constants, the same or different for each term. A specific case will most easily explain the method. Suppose that a division of θ into 73 parts and of ϕ into 73 parts gives a sufficient number of values of S for the purposes required. What we need is the value of S when θ, ϕ are any multiples of 360°/73. Put d for 360°/73. The

* Provided second differences be negligible.

† Unity is added to every sine or cosine so that the function to be treated shall be always positive. This plan is always adopted. In the analysis of observations, considered below, the same number is added to every observation for the same purpose.

method consists in finding S for $\phi = 0$ when $\theta = 0, d, 2d, \ldots 72d$; then for $\phi = d$, $\theta =$ same series; and so on up to $\phi = 72d$.

Since i, j are integers, and since $id + jd =$ multiple of d when multiples of 360° are rejected, it is evident that we only require the values of each term for the angles $i\theta + j\phi = 0, d, 2d, \ldots 72d$. Hence

$$A_{i,j}\{1 + \cos(p.d + \alpha_{i,j})\}$$

is first computed for each term with $p = 0, 1, 2, \ldots, 72$.

10. Suppose $i = 2$, $j = 3$. Let the 73 values of the term be entered on a tape containing 73 cards with its ends joined together, in the order $p = 0, 2, 4, \ldots 72, 74\ (=1), 3, 5, \ldots 71$. Number them in red ink 0 to 72 in the order in which they are written. Then one turn of the carrier is equivalent to increasing θ by d. If we start with the red number 0, we shall have the value corresponding to $\phi = 0$, and by turning the carrier we shall see the successive values for $\phi = d, 2d, \ldots 72d$.

When $\phi = d$, we have $j\phi = 3d$. We must therefore start with the red number corresponding to $3d$, that is, the number 38, and then successive turns give the values for $\theta = d, 2d, \ldots 72d$ with $\phi = d$. Similarly for all the values of ϕ.

The formation of these starting arguments is simple. For

$$\phi = 0, d, 2d, \ldots$$

we have

$$3\phi = 0, 3d, 6d, \ldots 72d, 2d, \ldots 71d, d, \ldots 70d,$$

giving the card numbers

$$0, 38, 3, 41, 6, 44, \ldots$$

that is, the remainders after the successive multiples of 38 have been divided by 73.

11. Tapes are made out for all the terms in a similar manner, the number 73 being the same for all. They are placed over the carrier, which is lowered on to the frame so that the hanging portions of the tapes fall between the guides as before. For $\phi = 0$, $\theta = 0$, all the card numbers are 0 and these are brought on to the face of the carrier. The 73 sums obtained after each turn of the carrier are the values of S for $\phi = 0$, $\theta = 0, d, \ldots 72d$. The tapes are then readjusted for the value $\phi = d$, $\theta = 0$, and the same process followed. The 73 card numbers required for each term at the beginning of each set of 73 sums are given to the operator at the start, so that he can form all the 73^2 sums without further supervision.

In practice some modifications have been made in the 21 tables computed in this way. It was not necessary in some of the tables to obtain S for all the values of ϕ; alternate values were then computed. It is evident also that all the terms for which i/j has the same value may be put together on one tape.

If there is a term for which $i = 0$, the tape for it is held fixed during each series of 73 sums. After the table of 73^2 sums has been completed, some interpolation and rearrangement are necessary to put it into the usual form required in lunar tables; this, however, is another question which need not be insisted on here.

12. The need for the number of parts into which θ, ϕ are divided to be prime to i and j is easily seen. For example, if we had used 68 instead of 73, the first 34 turns of the carrier in the term for $i = 2, j = 3$, would bring us back to zero. Thus only 34 values would be required for $\phi = 0$. When, however, $\phi = 360°/68$ the proper values would not appear on the tape at all. Similarly if the number is not prime to j.

13. The illustration exhibits the device arranged for this purpose. There are 13 tapes corresponding to the 13 terms which the double-entry table under treatment contains. The operator sits in front with the frame on his left so that he may follow the numbers on the carrier with his left hand while he operates the comptometer with his right hand. Also on the left is a Burroughs adding and printing machine on which each sum as it is obtained is recorded. This machine also records the sum of each series of p ($= 73$ in this case) sums. Since every value on each tape is used once and only once in each series, the sum of each series of sums should be always the same (unless there is a term for which $i = 0$). This furnishes a useful test. Errors are detected also by examining the differences. Duplicate calculation has not been necessary.

Analysis of Observations.

14. When the analysis of numerous observations at equal time-intervals into many harmonic constituents is required, as in tidal work, the labour of direction and computation can be reduced to equally simple rules. Suppose, for example, a year's observations of tide heights at hourly intervals is given * and the usual process for the analysis of observations be adopted. There will be nearly 8800 observations, and the work consists in forming sums in groups of these heights in as many different ways as there are harmonic terms to be found. The number of the sums to be obtained varies with the period of the term, but practically all the observations are used in every term. With the comptometer the actual work in the case of each term is the same, for there is no difference between the labour of finding 24 sums each of 350 numbers and that of finding 12 sums each of 700 numbers.

* This example is taken only for the purpose of illustrating the use of the device for the analysis of large numbers of observations. An attempt to discover whether there is any saving of expense in this special problem over that necessary with the use of the apparatus described by Sir George Darwin in *Proc. Roy. Soc.*, vol. 52 (1892), p. 345, would require a careful analysis and probably also a reconstruction of the general plan of reduction. I am much indebted to Sir George Darwin for criticisms on the method of presentation of the device and its applications, which led to this paper being almost entirely rewritten.

Let the observations be written on a tape long enough to contain them all, the successive cards being numbered 0, 1, 2, . . . preferably in red ink. The ends of the tape are not joined together. Hence, in order to prevent the tape from slipping off the carrier when any one of the first few or of the last few numbers appear on its face, about 20 cards at the beginning and at the end of the tape should be left blank. About 600 feet of it will be required.

Suppose that it is desired to analyse for a period of $25^h\ 49^m$. For this we obtain the average of the heights at the exact hours which are nearest to multiples of $25^h\ 49^m$, that is, at 0^h, 26^h, 51^h, 77^h, . . .,* then those an hour later in each case, and so on up to those which are 25 hours later. For this arrangement, the tape is wrapped over and over the carrier, not tightly but with long loops hanging down, there being 24 or 25 cards on each loop. There will then be some 350 cards on the face of the carrier and 350 loops. The operator is instructed to adjust the tape so that the red numbers 0, 26, 51, 77, . . . appear on the face of the carrier. No further instructions, except that of adding the observations thus presented, are necessary. After this is done and the sum recorded, he turns the carrier through 180°. This brings on to its face the observations at 1^h, 27^h, 52^h, 78^h, . . . which are similarly added and recorded. After 26 sums have been thus obtained, each sum is divided by 350 and we have the average values at 0^h, 1^h, . . . 25^h, of the harmonic term of period $25^h\ 49'$. These are analysed in the usual way in order to find the amplitude and epoch.

15. As one carrier bearing 350 loops is not a convenient arrangement the following modifications may be made in order to bring the work within the scope of a frame but little larger than that described above. If we analyse for a period of $3 \times 25^h\ 49^m = 77^h\ 27^m$, we really obtain the ordinates of the Fourier curve for

$$\Sigma A_i \cos(i\theta + \alpha_i) \qquad i = 0, 1, 2, \ldots,$$

where the period of θ is $77^h\ 27^m$. If there are no terms present except that of $i = 3$, the curve of averages will actually consist of ordinates for three successive periods of the term $A_3 \cos(3\theta + \alpha_3)$ which has a period of $25^h\ 49^m$. The first period starts at 0^h, the second at $25^h\ 49^m$, and the third at $51^h\ 38^m$. The ordinates between 25^h and 52^h must then be interpolated so as to get the values at hourly intervals from $25^h\ 49^m$ to $51^h\ 38^m$. The ordinates between 51^h, 78^h, are treated in a similar manner. We thus have three sets of values each of which starts at the same interval from the maximum; the average gives the 26 numbers from which the amplitude and epoch are deduced as before. The tides with periods

* If the exact period be expressed in decimals of an hour these numbers can be formed rapidly on an arithmometer. They are, however, the same for all ports.

approximating to 12 hours are similarly dealt with, but here we have six periods in the range.

With this method, the tape is wrapped round the carrier as before but so as to give rather less than 120 loops, there being 76 or 77 cards in each loop. The loops will hang below the carrier to a depth of 30 ins.—sufficient to keep their lower portions off the floor. The instructions to the operator then consist of the numbers 0, 77, 155, 232, . . . of the cards which are to appear on the face of the carrier at the start. He operates as before to obtain the 78 sums.

With this modification goes another, which simply consists in arranging the tape so as to show three columns of 40 numbers each instead of one of 120 numbers. Room on the frame is provided for several carriers side by side. If we take three, the tape can go 40 times over the first carrier, then 40 times over the second, then 40 times over the third, care being taken that the directions of wrapping are clockwise (or counter-clockwise) on all three. After the sum of the three columns has been obtained all three carriers are turned through 180°.

16. The labour of making 120 turns of the tape round the three carriers can be reduced to 40 turns, as follows:—Wrap the tape round one carrier so as to make 40 loops each of 232 or 233 cards, drop the loops between the guides, and lower on to the frame. Insert two other carriers underneath that already in position. Separate the three carriers so that they lie side by side on the frame. Then draw up each loop with the hand and drop a third of it between each pair of adjacent carriers, so that it will resemble the letter W. After adjustment of the tape according to the given table of numbers, the first carrier will have on its face the observations numbered 0, 232, 465, . . .; the second, those numbered 77, 310, . . .; and the third, those numbered 155, 387, . . . The only difference is that the same 120 numbers will be summed in three columns in a different order to the previous arrangement of three columns.

17. Various modifications of the mechanical arrangement of the tape will suggest themselves. With a slightly larger frame, six carriers, each bearing about 60 loops, could be used; the average ordinate at each hour of the period approximating to 25 or 26 hours could then be obtained directly. Or the tape might be divided into a few sections and the results in each section obtained separately.

18. After the summation for one period has been performed the operator has only to readjust the tape so that the cards appearing on the faces of the carriers bear the numbers proper to the new period and proceed as before. Once the tape has been placed over the carriers the readjustment is rapidly and easily done. If, in doing so, a loop becomes so short that much moving of the tape is necessary, it may always be dropped. A spare carrier is inserted underneath that bearing the short loop and pushed up to the loop. The original carrier is then withdrawn in the opposite direction

until the ends of the two carriers are separated, when the loop can be dropped without disturbing the other portions of the tape. The original carrier is then pushed back into place and the spare one withdrawn.

19. As a loop is not easily added, the terms should be taken in the order of the length of their periods, the shortest coming first. For very long periods, the lower parts of the loops may be resting on the floor; there is no real objection to this, if care be taken that they do not become entangled. Even this can be avoided by using other carriers to hold up the loops, the cards on the face of one carrier only being used in forming the sums.

20. The actual operation of the device has so far been limited to forming double-entry tables. The arrangements and preliminary calculations for summing numerous small harmonic terms are under way, and the work of summation will be started as soon as the twenty-one double-entry tables have been finished.

It may be stated that very little trouble has been experienced by cards becoming detached from the tape. The tapes so far used have each made between one and two hundred complete revolutions over the carrier, and would probably stand a thousand before becoming too much worn for further use.

21. If dimensions different from those described above be adopted, attention should be paid to the following details. In order to avoid a tendency to force the cards off the tape, the breadth of the carrier should be slightly less than the length of a card, and the distance between consecutive cards a little greater than the thickness of the carrier, the latter difference not to be less than the former. The distance between the guides should be a little greater than the width of the tape to prevent friction. A certain amount of "play" should be allowed for the guides in their sockets so that a slight variation in either the width of the tape or the distance between the guides may not cause trouble. For offices in which work of this nature is continuous, it is possible that more permanent forms of tape might be made, say of xylonite cards joined by small rings, from which the observations might be washed after the analysis has been completed.

Yale University:
1912 *February* 1.

A Proposal for the Comparison of the Stellar Magnitude Scales of the different Observatories taking part in the Astrographic Catalogue. Second Note. The Bordeaux Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The proposal referred to in the title was made in March 1909 (*M.N.*, lxix. p. 393), and submitted to the meeting of the Comité Permanent at Paris in April 1909, though it met with very little favour. It is simply this—

That the number of images recorded under each unit of the magnitude scale be counted and tabulated.

Illustrations showing the kind of information obtainable in this way were given from the Oxford, Greenwich, Potsdam, Toulouse, Helsingfors, and Paris results; and it was pointed out that there were evidences of systematic error in all these cases which could best be investigated at the observatory, since some of them might depend on conditions not recorded in the published volumes.

It was also pointed out that these counts did not support Kapteyn's conclusion that the Galaxy was relatively richer in faint stars than the non-galactic parts of the sky, and possibly this fact may be partly responsible for the scant favour accorded to the simple suggestion; for Professor Kapteyn told me in a friendly but frank manner that he could not bring himself to believe the conclusion. The committee, at any rate, decided in favour of a more elaborate investigation, by photographing the region of the Pole on the same plates (and at the same altitude) as selected portions of the zones of each observatory. This procedure is perfectly sound, and the only possible objection to it is that it will delay the completion of the main work, already lamentably in arrear at many observatories—unless the investigation of magnitude scale is itself deferred until the main work is completed. My own view is emphatically that the main work should take precedence, and that it is already more than high time that special measures should be taken to complete it at an early date: and it is therefore in no spirit of opposition to the decisions above quoted (for photographing the Pole, etc.), which I cordially approve, but simply in order to show what can be done pending their achievement, that I recall attention to the very simple procedure already advocated. The present note will show that it will provide valuable provisional information as to the systematic errors of a scale: and if these are ultimately assessed by an independent process, the counts will by no means be wasted, but will be available for the inverse problem of determining the distribution of different magnitudes over the sky.

2. The chief illustration of this paper is taken from the Bordeaux Tome I. published in 1905: zone $+17^{\circ}$. The counts were all made by the very kind volunteer help of Miss Riddle. The stars are classed in half magnitudes, by means of a scale constructed at the observatory from measures on the "Pritchard

types." The process is so simple that no explanation is necessary. The counts were made for each plate separately, and the totals checked with that for the whole plate: but it will be sufficient for the present purpose to tabulate the totals for each hour of R.A., which is accordingly done in Table I. At the foot are given—

First, the sum of each column by itself;
Second, the cumulative sum N′ of all stars up to and including that magnitude;
Thirdly, log N′, the quantity used by Kapteyn, or rather that quantity increased by a constant.

Now under any heading such as 8·0 are doubtless included stars from about 7·75 to 8·25, and thus the values of log N may be taken as corresponding to the headings increased by 0·25 in each case. We proceed to compare the totals with Kapteyn's formulæ.

TABLE I.

Number of Images recorded under each Half Magnitude in the Catalogue of Bordeaux +17° (Tome I.).

Mag. / R.A. h	6·0 and brighter.	6·5	7·0	7·5	8·0	8·5	9·0	9·5	10·0	10·5	11·0	11·5	12·0	Total.
0	3	5	9	12	31	60	106	123	207	187	181	129	88	1141
1	6	4	6	13	19	43	114	185	212	138	150	105	106	1101
2	8	1	4	8	20	42	88	148	185	137	149	65	87	942
3	2	5	5	13	29	52	93	149	240	197	237	175	76	1273
4	3	1	4	17	28	36	85	91	138	91	121	85	91	791
5	6	8	9	31	74	150	397	453	563	506	604	489	252	3542
6	9	1	25	30	91	194	441	514	717	699	927	733	419	4800
7	16	11	30	70	136	208	348	395	536	505	696	647	58	3656
8	15	10	23	37	80	138	203	235	336	292	300	252	35	1956
9	11	9	24	38	78	130	203	187	224	193	222	175	0	1494
10	7	13	18	24	74	80	130	141	187	118	125	90	0	1007
11	13	6	19	21	49	75	126	131	180	129	212	205	0	1166
12	12	7	13	18	42	56	88	102	177	150	235	357	0	1257
13	7	4	20	15	22	50	108	143	190	171	208	208	0	1146
14	9	3	18	25	48	78	111	107	145	121	155	126	0	946
15	9	7	15	15	47	54	111	128	223	207	307	384	8	1515
16	11	7	14	18	35	80	135	149	199	177	321	458	0	1604
17	39	19	29	64	114	163	326	339	446	409	449	436	1	2834
18	19	11	27	39	96	161	326	416	549	444	667	654	0	3409
19	24	12	31	49	124	196	430	518	724	735	876	715	34	4468
20	7	4	22	46	89	158	304	420	613	648	779	431	214	3735
21	6	1	9	15	37	95	230	309	407	442	484	416	169	2620
22	2	4	8	10	23	47	134	217	325	271	361	204	175	1781
23	4	3	4	16	23	39	132	198	261	190	243	190	260	1563
m	248	156	386	644	1409	2385	4769	5798	7984	7157	9009	7729	2073	49747
N′	248	404	790	1434	2843	5228	9997	15795	23779	30936	39945	47674	49747	
log N′	2·395	2·606	2·898	3·156	3·453	3·718	4·000	4·199	4·376	4·490	4·601	4·678	4·696	

3. In *Pub. Ast. Lab., Groningen*, No. 18, Table I., Kapteyn gives log N, the number of stars up to and including each half magnitude *per square degree*. Now we have added together the counts of 180 plates each of 4 square degrees and more. The plates have 26 spaces of 5′ each instead of 24; but the number of stars undoubtedly falls off at the edges, so that the equivalent homogeneous area is a little uncertain. For a reason that will presently appear, let us however consider the whole plate equally good: we must thus subtract from our log N′ to get Kapteyn's log N, the quantity

$$\log 180 \times 4 \times (13/12)^2 = 2{\cdot}927,$$

and the comparison is given in Table II.

TABLE II.

Comparison of Bordeaux Results with Kapteyn's Formulæ.

	Log N.				
Mag.	Bord.	K_0^{90}.	K_0^{20}.	$B-K_1$.	$B-K_2$.
6·25	9·468	9·185	9·312	+·283	+·156
6·75	9·679	9·435	9·568	+·244	+·111
7·25	9·971	9·683	9·822	+·288	+·149
7·75	0·229	9·928	0·074	+·301	+·155
8·25	0·526	0·170	0·324	+·356	+·202
8·75	0·791	0·409	0·572	+·382	+·219
9·25	1·073	0·646	0·819	+·427	+·254
9·75	1·272	0·880	1·065	+·392	+·207
10·25	1·449	1·112	1·308	+·337	+·141
10·75	1·563	1·340	1·548	+·223	+·015
11·25	1·674	1·565	1·787	+·109	−·113
11·75	1·751	1·788	2·022	−·037	−·271
12·25	1·769	2·010	2·254	−·241	−·485

4. Under the column K_0^{90} is given Kapteyn's general result for all galactic latitudes, and the excess of the Bordeaux result is given under the heading $B-K_1$. It will be seen that this is considerable until we reach about magnitude 10·5. It might be contended that since the Bordeaux zone crosses the Galaxy twice, and lies near it between 22^h and 4^h, we should use Kapteyn's numbers for nearer the Galaxy. Accordingly, under the heading K_0^{20} are given his numbers practically for the Galaxy itself, and it will be seen that the excess still exists. There can be no doubt that the brighter stars are assessed too bright at Bordeaux. Inverting the problem, let us find Kapteyn's magnitudes corresponding to the Bordeaux values of log N. We will take the column K_0^{90} as the best general approximation.

Table III.

Kapteyn's Maynitudes from Bordeaux Counts.

Bord. Mag.	log N.	K_0^{90}.	B–K.	Bord. Mag.	log N.	K_0^{90}.	B–K.
6·25	9·468	6·82	−0·57	9·75	1·272	10·59	−0·84
6·75	9·679	7·24	−0·49	10·25	1·449	10·99	−0·74
7·25	9·971	7·83	−0·58	10·75	1·563	11·22	−0·47
7·75	0·229	8·38	−0·63	11·25	1·674	11·50	−0·25
8·25	0·526	9·00	−0·75	11·75	1·751	11·64	+0·11
8·75	0·791	9·56	−0·81	12·25	1·769	11·68	+0·57
9·25	1·073	10·15	−0·90				

The column B – K gives the error of the Bordeaux magnitudes if we adopt Kapteyn's numbers as a standard. We need not stop to refine upon this comparison because of the systematic difference to which we now proceed.

5. Returning to Table I., the last column (of totals) reminds us that the zone $+17^\circ$ crosses the Milky Way about 6^h and 19^h. Let us take $5^h - 7^h$ and $18^h - 20^h$ as representing the neighbourhood of the Galaxy: and let us take the two portions $23^h - 4^h$ and $8^h - 17^h$ as representing non-galactic regions, adding together the hourly results to form four groups as in Table IV. It will suffice for our present purpose to retain only a limited number of values of log N.

Table IV.

Values of log N′ for Groups.

Bord. mag.	6·75	7·75	8·75	9·75	10·75	12·25
Kapt. mag.	7·24	8·38	9·56	10·59	11·22	11·68
Non. 21^h– 4^h	1·76	2·32	2·92	3·51	3·83	4·05
Gal. 5^h– 7^h	1·71	2·39	3·04	3·56	3·86	4·08
Non. 8^h–17^h	2·34	2·84	3·34	3·73	3·98	4·17
Gal. 18^h–20^h	1·89	2·46	3·05	3·55	3·86	4·06

From the last column we see that the four groups contain very nearly the same number of stars altogether. If 0·10 be subtracted from all the numbers for the third group, the last two columns become nearly uniform. But there is a considerable excess of bright stars in the third group, though not in the first.

6. If this excess is real, it means a difference between the two sides of the Galaxy, *not* an effect of the Galaxy as compared with the rest of the sky. Now it may quite possibly be real. In Stratonoff's *Études sur la structure de l'univers* (Tachkent, 1900), there is a patch of stars of 6·0 mag. and brighter close to the N. Pole of the Milky Way, near which the Bordeaux zone passes. But the excess is not kept up in following maps, and the figures generally are not such as will accord with the Bordeaux figures.

It seems more probable (although the idea of a real physical difference is not to be altogether excluded) that there is a systematic error of a seasonal character in the Bordeaux results, and we shall proceed on this supposition for the present. Such an error would appear chiefly as a harmonic of the first order, and would thus be readily distinguishable from a galactic effect, which would show chiefly in the harmonic of the second order (since the Galaxy crosses the zone twice). One obvious physical cause for such an effect is temperature. It has been found that plates are more sensitive in cold weather, and with a fixed time of exposure there should therefore be more faint stars on the plates in the winter. But, keeping an open mind as to the cause, let us analyse the values of log N at the points indicated in Table IV.: viz. intervals of a whole magnitude on the Bordeaux scale, with the exception of the last.

[On reference to Table I. it will be seen that the faintest stars on the plate sometimes appear as 12·0 and sometimes as 11·5 only, and for the final column it is more satisfactory to include *all* the stars on the plate.]

TABLE V.

First Harmonics of log N′, *in terms of R.A.* (α).

Mag.		Coeff. of		Referred to 12ʰ·25.		Log N′ for all Stars.
B.	K.	sin α.	cos α.	sin α.	cos α.	Combined.
6·75	7·24	−0·12	−0·19	−0·07	−0·23	+0·24 cos (α − 197°)
7·75	8·38	− ·08	− ·17	− ·03	− ·21	+ ·21 cos (α − 188°)
8·75	9·56	− ·04	− ·13	+ ·01	− ·17	+ ·17 cos (α − 177°)
9·75	10·59	− ·02	·00	+ ·03	− ·04	+ ·05 cos (α − 140°)
10·75	11·22	− ·03	+ ·04	+ ·02	·00	+ ·02 cos (α − 90°)
12·25	11·68	− ·05	+ ·04	·00	·00	

7. From the third and fourth columns it will be seen that the effect is small for the faint stars, but large for the bright ones. What chiefly concerns us is the *relative* effect, and we may accordingly subtract the last coefficients from all the rest, as shown in the fifth and sixth columns. We see that there is a range of 0·48 in the differences of log N′.

Now compare with this the effect ascribed by Kapteyn to the Galaxy. His approximate magnitudes are given in the second column and his extreme values of log N are

		$b=0°$.	$b=90°$.	0°−90°.
For mag.	7·25	9·88	9·51	+0·37
,,	11·75	2·14	1·38	+0·76
	Difference	−2·26	−1·87	−0·39

8. Hence the systematic differences we have discussed above are greater (0·48 as compared with 0·39) than the extreme effect

between the Galaxy and its Pole, as determined by Kapteyn. It is clear that before we can arrive at trustworthy conclusions about the effect of the Galaxy, we must investigate this systematic effect, whatever its origin may be. This discussion can obviously be best undertaken at the observatory, and it does not seem necessary to wait for the more elaborate determinations of magnitude proposed at the last meeting of the committee. On the contrary, there may be advantages in delaying this extra work until it is known what is the source of the differences noted; for this knowledge may suggest a particular arrangement of the control investigation. By way of illustration, suppose it is a temperature effect: then there will clearly be advantages in repeating the control plates (or some of them) at widely different temperatures.

9. Before attempting to assign the origin, however, we must remove a small part which would be due to the Galaxy according to Kapteyn. It is easy to find this by writing down the galactic latitude of each hour, and taking out from his tables the difference in log N for magnitudes (say) 7·0 and 12·0. Analysing these numbers we get for the part of the first harmonic due to the Galaxy, at most

$$-0{\cdot}02 \sin a - 0{\cdot}06 \cos a,$$

leaving unexplained in the first row of Table V.

$$-0{\cdot}05 \sin a - 0{\cdot}17 \cos a.$$

It is noteworthy that these expressions are the same in form, and in the numerical ratio of 3 to 1; in other words, it would appear that we could account for the first harmonic by increasing Kapteyn's galactic effect in the ratio of 4 to 1. But apart from the general improbability of such a magnification, there is the objection that the chief effect of the Galaxy should appear in the second harmonic and does not, as we see below.

10. The advantage of working with harmonics is that one harmonic does not affect another. The existence of this systematic effect, whatever it may be, will not affect the second harmonic, which we can calculate without removing the first. (Were it not for this, it would be simpler to work directly in terms of galactic latitude.) Let us first see what the second harmonic should be with Kapteyn's numbers. Taking the differences of log N for magnitudes 7·0 and 12·0 as above, at the galactic latitudes corresponding to the beginning of each hour, we find for the second harmonic,

$$\log N_7 - \log N_{12} = \text{const.} + {\cdot}05 \sin 2a + {\cdot}17 \cos 2a$$
$$= + {\cdot}18 \cos 2(a - 8^\circ).$$

The extreme magnitudes with which we deal in Table VI. are 7·24 and 11·68, differing by 4·42 magnitudes instead of 5·0. Hence we must reduce the coefficient to $+{\cdot}18 \times {\cdot}9 = +{\cdot}16$.

Now analysing the observed values of log N′ just as they stand, we get the third and fourth columns of Table VI., showing the effect

of the Galaxy on all magnitudes. But what concerns us at present is the extreme *differential* effect, and we therefore subtract the values for 12·25 from all the others as in columns 5 and 6.

TABLE VI. Bordeaux +17°.

Second Harmonics of log N′, in terms of R.A.

Mag.		Coeff. of		Referred to log N′ for all Stars.		
B.	K.	sin 2 α.	cos 2 α.	sin 2 α.	cos 2 α.	Combined.
6·75	7·24	−·13	−·15	+·05	+·07	+·086 cos 2 (α − 18°)
7·75	8·38	−·16	−·17	+·02	+·05	+·054 cos 2 (α − 11°)
8·75	9·56	−·20	−·18	−·02	+·04	+·045 cos 2 (α + 13°)
9·75	10·59	−·17	−·20	+·01	+·02	+·022 cos 2 (α − 13°)
10·75	11·22	−·19	−·20	−·01	+·02	+·022 cos 2 (α + 13°)
12·25	11·68	−·18	−·22	·00	·00	·000

The effect is thus of the kind indicated by Kapteyn, but little more than half the amount. In view of the systematic error above noticed, we must not lay too much stress on this result until the source of the error is identified: but it is noteworthy that all the evidence hitherto obtained should be in the direction of reducing or annulling the effect of the Galaxy, as indicated by Kapteyn, on the relative number of bright and faint stars.

11. Returning to the error itself, it is worth while considering how far it can be a direct temperature effect. The temperature at which the plates were taken is given in the Introduction to the volume, and the mean for each hour is approximately as shown in Table VII.

TABLE VII.

Temperature (C.) and Mean Date for Mean Plate of each Hour.

(Bordeaux +17°.)

R.A.	Temp.	Date.	R.A.	Temp.	Date.	R.A.	Temp.	Date.
h	°		h	°		h	°	
0·5	10·1	Oct. 23	8·5	6·7	Mar. 6	16·5	16·4	June 18
1·5	6·4	Nov. 17	9·5	8·5	Mar. 16	17·5	17·8	July 6
2·5	6·4	Nov. 31	10·5	7·7	Mar. 11	18·5	20·3	July 16
3·5	1·0	Dec. 9	11·5	8·8	Apr. 5	19·5	19·9	Aug. 7
4·5	1·3	Jan. 2	12·5	9·1	Apr. 15	20·5	19·0	Aug. 22
5·5	1·8	Feb. 6	13·5	12·8	May 11	21·5	15·0	Oct. 19
6·5	5·7	Feb. 18	14·5	17·3	May 31	22·5	14·0	Oct. 16
7·5	6·4	Feb. 27	15·5	15·1	June 1	23·5	12·5	Nov. 4

Now the first harmonic is

$$-15^{\circ}\cdot6 \sin \alpha - 0^{\circ}\cdot2 \cos \alpha = 15^{\circ}\cdot6 \cos (\alpha - 269^{\circ}),$$

which has its maxima close to the zero-points of the effect exhibited in Table V. It seems therefore quite improbable that the effect is a temperature effect, and its origin must be left quite an open question, which can be best investigated at the Bordeaux Observatory.

12. Though not connected with the temperature it may still be connected with the date, and accordingly Miss Bellamy * has filled in the mean date for each hour from information given in the volume. For instance, we have to explain why bright stars should be less numerous about 0^h than about 12^h; or what is equivalent, why they should be estimated brighter. Now wind, by shaking the telescope, might enlarge the diameter of the bright stars; and if there is more wind in November than in April and May, this might account for the observed effect. Wind may produce an effect of the kind noted in two ways: it enlarges the diameters of bright stars and makes them appear brighter, while at the other end of the scale it cuts out the faintest stars, the images of which wander about and fail to impress the plate at any point. Bright stars are made to appear brighter, and faint stars fainter.

13. It is natural to inquire whether the magnitudes assigned at other observatories show systematic errors of this kind. At present no other zone has been completely counted in this way except the Oxford zones; and attention has been mainly concentrated hitherto on the differences between different measurers of the Oxford plates. But a random sample seems to show that there is no difference of the same *size* in this case. Take, for instance, the measures made by T.M. in zone $+29°$. The values of log N′ down to various limits of measured diameter (d) are as follows:—

R.A.	$d>29$.	>21.	>14.	>8.	All Stars.
8^h—16^h	2·36	2·71	3·13	3·59	3·69
22^h—2^h	2·40	2·77	3·15	3·59	3·83
Difference	−·04	−·06	−·02	·00	−·14

14. But if we are content to neglect the differences of personality and take the counts for a couple of zones as they stand; and if further we take all images measured of the same diameter together (neglecting the fact that the quality of the plates varies), then we can tabulate the harmonics for log N′ as follows:—

First Harmonics for Oxford Zones $+26°$ and $+31°$.

d.	Approx. Mag.	Sin α.		Cos α.		Combined.	
		$+26°$.	$+31°$.	$+26°$.	$+31°$.	$+26°$.	$+31°$.
>50	6·5	−·090	−·085	−·005	+·093	·091 cos $(\alpha+93°)$	·126 cos $(\alpha+42°)$
>40	7·5	−·078	−·078	+·084	+·085	·114 ,, ,, 43	·115 ,, ,, 43
>30	8·5	−·063	−·082	+·080	+·083	·102 ,, ,, 39	·117 ,, ,, 45
>22	9·5	−·094	−·067	+·052	+·103	·107 ,, ,, 61	·123 ,, ,, 33
>15	10·5	−·106	−·082	+·081	+·081	·133 ,, ,, 53	·115 ,, ,, 45
All stars	12·0	−·134	−·156	+·137	+·156	·192 ,, ,, 45	·220 ,, ,, 45

* Incidentally she noticed that there are discrepancies in the dates given for the same plate on pp. 66–67, 84–85, 159, 215–16, 236–67. They can usually be corrected by inspection of neighbouring headings; but it is not clear from the headings alone whether cliché 377 was taken in 1893 as on p. 66, or in 1899 as on p. 67.

15. It will be seen that the coefficients are fairly steady down to the tenth magnitude at any rate. Excluding the last line, the mean coefficient for $+26°$ zone is ·105 and for $+31°$ zone ·119. Also the epoch is pretty steady, especially in zone $+31°$, which represents later and perhaps more uniform measures. The mean epoch is $-55°$ for zone $+26°$, and $-42°$ for zone $+31°$: mean of both, say $-48°$. The interpretation here is chiefly that the Milky Way is far richer at $19\frac{1}{2}^h$ than at $5\frac{1}{2}^h$ where it cuts the Oxford zones. The maximum given by the formula falls at $20^h\ 45^m$ and the minimum at $8^h\ 45^m$, but the significance of this difference in epoch need not be discussed here. Of this difference, which is essentially a difference of distribution in *longitude*, Kapteyn's distribution (in galactic latitude only) can take no account. There is, however, a first harmonic resulting from his distribution, owing to the fact that the Galaxy does not cut the Oxford zone symmetrically. For zone $+30°$ the first harmonics are for

$$\text{mag. } 7\cdot0 \quad +\cdot018 \sin\alpha + \cdot094 \cos\alpha = \cdot096 \cos(\alpha - 11°);$$
$$\text{mag. } 12\cdot0 \quad +\cdot040 \sin\alpha + \cdot209 \cos\alpha = \cdot211 \cos(\alpha - 11°).$$

16. The coefficients are not very different from those found above, but

(1) The epoch differs by $48° + 11° = 59°$, owing to the fact that Kapteyn ignores any effect in galactic longitude.

(2) The rise in the coefficient, which in Kapteyn's tables is gradual, appears in the Oxford results comparatively suddenly near magnitude 10 or 11.

17. Coming to the second harmonic, the figures obtained from Kapteyn's table, declination $+30°$, are

$$\text{mag. } 7\cdot0 \quad -\cdot040 \sin 2\alpha - \cdot107 \cos 2\alpha = \cdot110 \cos(2\alpha + 160°);$$
$$\text{mag. } 12\cdot0 \quad -\cdot084 \sin 2\alpha - \cdot225 \cos 2\alpha = \cdot240 \cos(2\alpha + 160°).$$

For comparison with these we have

Second Harmonics for Oxford Zones $+26°$ and $+31°$.

d.	Approx. Mag.	Sin 2 α.		Cos 2 α.		Combined.	
		+26°.	+31°.	+26°.	+31°.	+26°.	+31°.
>50	6·5	−·145	−·113	−·183	−·178	233 cos (2α + 142°)	·211 cos (2α + 147°)
>40	7·5	−·138	−·145	−·173	−·182	·221 ,, ,, 141	·232 ,, ,, 141
>30	8·5	−·141	−·125	−·190	−·173	·237 ,, ,, 143	·214 ,, ,, 144
>22	9·5	−·162	−·167	−·195	−·160	·253 ,, ,, 140	·231 ,, ,, 134
>15	10·5	−·161	−·167	−·187	−·191	·247 ,, ,, 139	·254 ,, ,, 139
All	12·0	−·173	−·225	−·212	−·182	·274 ,, ,, 140	·289 ,, ,, 129

Neither coefficients nor epochs fit Kapteyn's figures at all well. There is a slight rise in the coefficients, but it cannot be put higher than ·015 per magnitude or ·075 for 5 magnitudes, about half

that shown by Kapteyn's figures. The mean epoch for zone $+26°$ is $141°$, and for zone $+31°$ is $139°$, in very good agreement and both differing sensibly from Kapteyn's $+160°$.

18. The important point about the features above noticed is that, if real, they are very *local*. The excess of bright stars about $12^h + 17°$, shown in the Bordeaux results, is not shown in the Oxford results for zone $+26°$. Again, the superior richness of the Milky Way at 19^h, shown by the Oxford results, does not appear in the Bordeaux results. This richness at 19^h is shown very clearly by Argelander (see Mr. Bellamy's diagram, *Monthly Notices*, vol. lx. p. 16) and therefore by Stratonoff. But it seems to be a local richness of the Milky Way.

Summary.

§ 1. Recurrence to the simple proposal made at Paris in 1909, which may save delay in completing the Astrographic Catalogue.

§ 2. Summary of counts for Bordeaux Dec. $+17°$ in hours of R.A. (Table I.).

§§ 3, 4. Approximate error of Bordeaux magnitude scale ranging from $-0·8$ to $+0·6$ of a magnitude, according to Kapteyn (Table III.).

§§ 5–8. There is also a systematic excess of bright stars, near R.A. 12^h.

§ 9. Which is four times Kapteyn's galactic effect.

§ 10. The second harmonic is only one-half that given by Kapteyn's figures. This is not necessarily error in Bordeaux, for other observatories do not support Kapteyn's figures.

§ 11. The error indicated by first harmonic is probably not due to temperature.

§ 12. Is it perhaps due to wind?

§§ 13–17. It is not in the Oxford measures, which, however, do not agree with Kapteyn's figures. The first harmonic is chiefly due to an inequality in galactic longitude, which he ignores; the second does not support his increased richness in faint stars, at any rate, not to the same extent.

A Tentative Explanation of the "Two Star Streams" in Terms of Gravitation. Second Note. The Position of the Centre of our System. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the preliminary sketch of last month it was pointed out how the appearance of dual convergence of stellar motions might result, not from the meeting of two streams of parallel motions, but from the convergence and divergence of stars visiting the gravitational centre of our system. On this hypothesis the centre should lie in the direction of one or other of the opposite vertices of movement, fixed by Mr. Eddington in R.A. 94°·2, Dec. +11°·9, and its opposite R.A. 274°·2, Dec. −11°·9, "very near to the position which Kapteyn originally announced." But no serious attempt was made in the former paper to determine which of these two alternatives was to be preferred, the work done being limited to an estimate of the average effect of displacement of the viewpoint from the centre. The preference vaguely felt was for the 18^h vertex, owing to the fact that the writer's attention has necessarily been specially drawn to the greater number of stars shown in this direction by the *Oxford Astrographic Catalogue*: and it was not for some little time that this vague predilection was overcome by the gradual accumulation of evidence in favour of the opposite vertex. It is the object of the present note to sketch the nature of this evidence, as furnished by materials already available; and to indicate how further tests may be made when we have a better knowledge of stellar distribution and velocity.

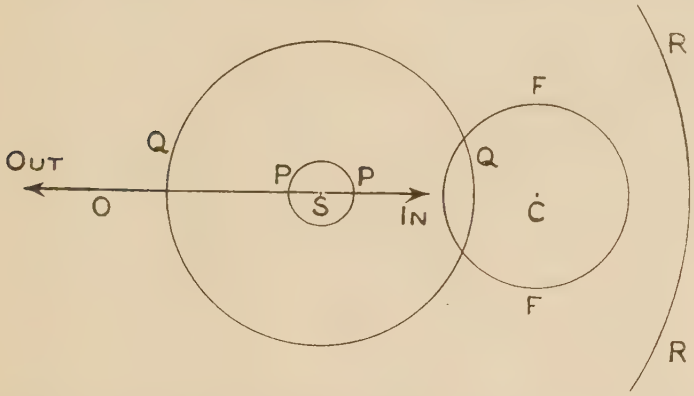
2. We will begin by an examination of stellar distribution. In the former note a hypothetical constitution for our system was suggested, built up of separate systems of stars, each of uniform intrinsic brilliancy, and distributed according to Schuster's law

$$\text{density} \propto c^5/(c^2+r^2)^{\frac{5}{2}}$$

where r is the distance from the gravitational centre, and c a constant for each system. This law is only strictly applicable when c is the same for all the systems: for only in that case would the law of attraction on any star retain its appropriate form. We were led to assume that c increased with intrinsic brightness: so that intrinsically faint stars would be accumulated near the centre, the intrinsically bright ones being widely diffused. The condensed systems would thus attract the diffused with a law approximating more nearly to that of a "central sun"—inversely as the square of distance from centre—which would modify the law of distribution. There is thus much that requires more elaborate investigation before anything like a final constitution can be proposed. But several considerations make it probable that the tentative constitution is sufficiently good to give us a first approximation. Firstly, the change of c at which we arrived is not very rapid. For

a difference of 1·83 (intrinsic) magnitudes c was only altered in the ratio 1·44 : and thus over the range of half a dozen magnitudes (which practically limits our present knowledge), c would only increase from 1 to 4. Secondly, the fainter stars are probably of smaller mass, and this will lessen the importance of their disturbance. We shall therefore proceed with the hypothetical system sketched in the last note.

3. But before proceeding to an exact integration, let us make an elementary examination of the kind of effect which displacement of the view-point from the centre will produce. One such effect was considered in the former paper: viz. the loss of intrinsically faint stars. But we then did not discriminate between different directions: it is now our object to discriminate; and especially to see in what particular the direction of the centre



differs from the opposite direction. For brevity we shall use the terms In and Out for these directions.

4. Thus, in the diagram, if S be the Sun (or Earth) and C the centre, the direction S C is In, the direction S O is Out. For simplicity, suppose that there are two systems of stars only, those intrinsically brighter (B) being so widely diffused that we may consider the distribution uniform and infinite, those intrinsically fainter (F) being confined to the sphere F F. From what precedes it will be seen that this is a crude qualitative representation of our hypothetical system.

Now consider stars of the (apparent) first magnitude, as observed from S. To appear of the first magnitude the F stars must be very close to S, say within the sphere P P. But there are none, since we have confined them to the central sphere F F. Hence the stars of the first magnitude will all be B stars, uniformly distributed, and will not help us to find the centre.

To appear of the second magnitude, the F stars need not be so near S: it will suffice if they are within the sphere Q Q say.

Now this does contain a few F stars, but only in the In-direction. Hence second magnitude stars will show a slight excess in the sense In minus Out.

To appear of the third magnitude, the F stars need only be within the sphere R R. This includes, near the In-direction, all the F stars: and there is thus a marked excess over the Out-direction, which still has none.

The criterion for identifying the centre is thus that there should be an excess of faint stars in that direction, after a certain magnitude is reached.

5. We will give greater precision to this crude analysis later. But in the present state of our knowledge it is almost sufficient in this form to proceed with the argument. If we had an exact comparison of the numbers of stars for each magnitude seen in the directions of the two vertices, we could at once see which was In and which Out. But, firstly, there is some uncertainty in our absolute magnitude scales; and there is special difficulty in comparing the north and south hemispheres. Secondly, the two vertices are in the Milky Way, and we are liable to error from local features of that irregular appendage to the system. (See, for instance, the previous paper on the Bordeaux magnitudes, p. 473.) To minimize systematic error it seems desirable to work in zones of declination; and to get as near as possible to the two vertices it is best to take zones near the equator. Hence, let us take the zone -5° to $+5^\circ$ on Stratonoff's maps—one from his maps of the northern and one from those of the southern hemisphere. Adding all the numbers for each hour, and finding the first harmonics (which are what concern us), we get

TABLE I.

Stratonoff's Zones, -5° to $+5^\circ$.

Mag.	1st Harmonic.
To 6·0 . . .	$1{\cdot}3 \cos (\alpha - 36^\circ)$
6·1 to 6·5 . . .	$1{\cdot}3 \cos (\alpha - 29^\circ)$
6·6 to 7·0 . . .	$1{\cdot}2 \cos (\alpha - 273^\circ)$
7·1 to 7·5 . . .	$0{\cdot}3 \cos (\alpha - 108^\circ)$
7·6 to 8·0 . . .	$0{\cdot}8 \cos (\alpha - 73^\circ)$
8·1 to 8·5 . . .	$1{\cdot}8 \cos (\alpha - 79^\circ)$
8·6 to 9·0 . . .	$2{\cdot}1 \cos (\alpha - 80^\circ)$
9·1 to 9·5 . . .	$1{\cdot}1 \cos (\alpha - 77^\circ)$.

The coefficients of the terms need not be scrutinized very closely: they represent variations in an average density of 10, the greatest range being thus 12 to 8. The important point for us is the direction in which there is an excess of stars, as shown by the epoch, and for the fainter stars this is steadily near 80° R.A. For the brighter stars it is more uncertain, *e.g.* for 6·6 to 7·0 it is at the opposite point. With the above criterion before us, we can have

no hesitation in fixing on the 94° vertex as the In-vertex (the direction of the centre), so far as this evidence goes. And we shall probably be right in ascribing the outstanding differences to disturbance by local features of the Milky Way. Thus it was remarked in the paper on the Bordeaux magnitudes (p. 473) that there is a considerable condensation of stars near 20^h in the Milky Way. From Mr. Bellamy's diagram (*M. N.*, lx. p. 16) the maximum is near $20^h + 40°$: and the Oxford figures, quoted on p. 472, show that it is an increase of both bright and faint stars alike. It dies out very quickly, not appearing in the Bordeaux zones; but still it will be likely to draw the maximum (due to the In-vertex) towards itself, and this might easily account for our finding 80° instead of 94° for the position in R.A. of the maximum.

6. But it is disappointing that we are not yet in a position to carry this scrutiny to fainter magnitudes. It was hoped that the Astrographic Charts might have supplied valuable evidence, but it was soon found that they were as yet far too incomplete for the purpose. On examining our stock of Charts near the equator, it was thought at first that the Algiers zone $-1°$ afforded just the material needed; but, unfortunately, there is a wide gap in at least one of the most critical portions: we have as yet received no maps in 19^h R.A., only three in 18^h, two in 20^h, three in 21^h, and two in 22^h. Moreover, near the other vertex 5^h and 7^h are scrappy. Hence at present very little can be said of fainter magnitudes. Analysing such figures as are available, *i.e.* the totals given on the Algiers Charts, we find the first harmonic to be

$$200 \cos (\alpha - 52°),$$

and, since the average number of stars is about 900, on Stratonoff's scale of 10 this would rank as $2{\cdot}2 \cos (\alpha - 52°)$, which is distinctly in favour of the 94° vertex. There is no other zone of maps in anything approaching completeness for this purpose. The beautiful maps of Max Wolf and Palisa may probably be utilised, but the time of exposure varies greatly, and much labour will be necessary to extract the information from them. At present, therefore, we can only indicate the direction in which further information must be sought, without attempting to supply it.

7. We must now supplement the crude analysis above used by a more exact investigation. Let us suppose in the first instance that we have a single system, in which the number of stars in unit volume at distance r from the centre is

$$nc^5/(c^2 + r^2)^{\frac{5}{2}}.$$

Let I be the intrinsic brightness of each star. Then the number of stars brighter than a given magnitude m, as seen from the centre, is determined by the distance ρ_m at which these appear of magnitude m. Thus

$$I/{\rho_m}^2 = 10^{-m \times 0{\cdot}4}$$

or

$$\log \rho_m = 0{\cdot}5 \log I + 0{\cdot}2\, m.$$

The number N seen in any direction within an element of angular area $d\omega$ is

$$N = nd\omega \int_0^{\rho_m} \frac{c^5 r^2 dr}{(c^2 + r^2)^{\frac{5}{2}}}.$$

Putting $r = c \tan \phi$, and $\rho_m = c \tan \phi_m$,

$$N = c^3 nd\omega \int_0^{\phi_m} \tan^2 \phi \sec^2 \phi \cos^5 \phi d\phi = \frac{1}{3} c^3 nd\omega \,.\, \sin^3 \phi_m \,.$$

Now suppose the view-point is at a distance a from the centre, and we are looking in the Out-direction. Let r as before be the distance from the view-point (Sun or Earth), and R that from the centre. Then

$$R = r + a = c \tan \phi$$
$$dr = c \sec^2 \phi d\phi \text{ as before}$$
$$N = nd\omega \int \frac{c^5 r^2 dr}{(c^2 + R^2)^{\frac{5}{2}}}$$
$$= cnd\omega \int (c \tan \phi - a)^2 \sec^2 \phi \,.\, \cos^5 \phi d\phi$$
$$= \frac{1}{3} c^3 nd\omega \left[\sin^3 \phi (1 - \frac{a^2}{c^2}) + \frac{2a}{c} \cos^3 \phi + 3 \frac{a^2}{c^2} \sin \phi \right]$$

and the limits are from $r = 0$ to ρ_m, or

$$\tan \phi_0 = a/c \text{ to } \tan \phi_m = (\rho_m + a)/c.$$

If we look in the In-direction

$$R = a - r \text{ as far as the centre and } r - a \text{ beyond.}$$

But only R^2 enters into the integral, and this is the same in both cases. Hence put

$$r - a = c \tan \phi \qquad : \qquad R^2 = c^2 \tan^2 \phi$$

where ϕ is negative for small values of r. We therefore simply change the sign of a throughout. The integral will take the same form, except that the term

$$\frac{2a}{c} \cos^3 \phi$$

will be negative, and the lower limit for ϕ is negative.

8. For purposes of calculation it is easiest to retain the auxiliary angle ϕ throughout, and tabulate values of the integrals for different values of a/c. We shall thus find the difference between the In- and Out-directions, which are obvious maxima and minima for all systems. The integrals for intermediate directions

are more troublesome, though they can be expanded in a series of harmonics. But the calculation for these extreme directions is simple. Thus let us put $a/c = 1$, so that the integral becomes

$$\pm\ 2\cos^3\phi + 3\sin\phi,$$

and the lower limit $\phi_0 = \pm 45^\circ$: and let us calculate the integrals for the values of ρ_m used in the formation of Table II. of the former paper, starting with $\rho_m{}^2/c^2 = \cdot 10$, and increasing ρ_m in the ratio corresponding to successive magnitudes.

TABLE II.

Calculation of Out-direction (away from centre).

ρ_m/c	tan ϕ	ϕ	3 sin ϕ	2 cos $^3\phi$	Sum.	Out-Intl.	Centre.	In-Intl.
0·00	1·00	45°0	2·121	0·708	2·829	...	...	...
0·32	1·32	52·9	2·393	0·440	2·833	·004	·027	·009
0·50	1·50	56·3	2·496	0·342	2·838	·009	·090	·047
0·79	1·79	60·8	2·619	0·232	2·851	·022	·239	·333
1·25	2·25	66·0	2·741	0·135	2·876	·047	·478	1·727
2·00	3·00	71·6	2·847	0·063	2·910	·081	·718	4·242
3·16	4·16	76·5	2·917	0·026	2·943	·114	·865	5·403
5·00	6·00	80·6	2·960	0·009	2·969	·140	·946	5·709
7·94	8·94	83·6	2·981	0·003	2·984	·155	·980	5·791
∞	∞	90·0	3·000	0·000	3·000	·171	1·000	5·827

The first six columns give the calculation, and the result for the Out-direction is in the seventh column. In the eighth and ninth are placed for comparison the corresponding results for the centre as a view-point, viz. [$\sin^3\phi$], and those for the In-direction. It will be seen that at the distance $a = c$ there is at first a deficiency, as compared with the centre, for both In- and Out-directions, and for the Out-direction this deficiency remains throughout, but for the In-direction the deficiency vanishes when $\rho_m = c$, and then becomes an excess of about six times. The ratio of In to Out rapidly rises to about 40 or 50 to 1, which could not fail to be noticed if it occurred in nature undiluted. But the superposition of systems reduces the effect considerably.

9. The most convenient way of tabulating the results is to form the logarithm of the ratio to the corresponding numbers for central view, as in the following Table III. The first column shows the values of r/c used in the former note, increasing in the ratio corresponding to one magnitude. The next four columns refer to view in the Out-direction, and a/c is given the negative sign. There is a steady defect for all systems. The next four give results for the In-direction, showing an excess followed by a

defect. The defect does not appear in rows after the fourth, because the table is not extended to large enough values of a/c.

TABLE III.

Excess of log N *for View-point at Distance* a *from Centre over the value for Central View.*

a/c. / r/c.	Out.				In.			
	−2·00.	−1·00.	−0·50.	−0·25.	+0·25.	+0·50.	+1·00.	+2·00.
·32	9·04	9·17	9·78	9·90	0·04	0·08	9·52	9·00
·50	8·64	9·00	9·66	9·82	0·10	0·12	9·72	8·78
·79	8·40	8·96	9·57	9·76	0·16	0·29	0·14	9·24
1·25	8·42	9·00	9·50	9·73	0·21	0·40	0·56	9·84
2·00	8·46	9·05	9·51	9·74	0·23	0·45	0·77	0·83
3·16	8·55	9·12	9·55	9·76	0·22	0·44	0·80	1·26
5·00	8·63	9·17	9·57	9·76	0·21	0·43	0·78	1·27
7·94	8·70	9·20	9·58	9·77	0·20	0·42	0·77	1·25
∞	8·78	9·24	9·59	9·77	0·20	0·41	0·77	1·25

10. Using this table, a calculation was made of the number of stars of different magnitudes, and the results were as follows:—

TABLE IV.

N = *Number of stars.*

	Out.	Centre.	In.	$\log N_{(in)} - \log N_{(out)}$.
Mag. 1	5·1	7·3	6·7	+0·12
2	12·8	21·8	18·6	+0·16
3	28·6	65·3	53·0	+0·27
4	65	195	158	+0·39
5	126	585	429	+0·53

More is not given, because experience suggested that the tables had not been extended far enough, and a new calculation will be made on a more elaborate scale; but there is sufficient to show the main features of the result—a defect of both In and Out as compared with the centre, and an excess for In as compared with Out. The star-ratio for the Out-direction is sensibly less than 3, and to suit observed facts we must probably modify the original hypothesis of the last note, as foreshadowed in § 34 of the note itself. This is an additional reason for deferring the complete discussion of this point. But the matter is now merely a question of arithmetic.

11. Let us now consider another piece of evidence pointing in the same direction in a remarkable manner. In his great memoir on the F. G. W. Struve double stars (*Mem., R.A.S.*, vol. lvi.) Mr.

Lewis has tabulated the ratio F/M of "fixed" to "moving" pairs in different parts of the sky. His suggestion is

> There is no apparent reason why distant stars should not have close companions endowed with orbital motion as frequently as those nearer to us, nor is there any known reason why double stars should not be evenly distributed in space. If we find this distribution is not uniform, it may be that in certain regions the stars are, as a whole, more distant from us than in others. Granting this, the proportion of fixed pairs to pairs showing relative motion should be *a function of* the distance of the stars from us.

12. The italics (mine) are simply to call attention to the fact that Mr. Lewis has not committed himself to a particular form of the function, though he has used the first power. At the moment I do not feel sure that a better supposition might not be made, but it will suffice at present to take Mr. Lewis's argument as it stands. He proceeds to sketch the dimensions of the universe of double stars as thus revealed.

> The points brought out seem to show that the stars around us form a universe very much the shape of an egg, and that we are not situated in the centre. The longer diameter may be roughly estimated as equivalent to 600 light years, and the smaller to 300 light years.

13. He gives a diagram of the section in R.A. with his suggested ellipse, $2\frac{3}{4}$ in. $\times$ $1\frac{5}{8}$ in., so that 1 in. corresponds to 200 light years. The centre of the ellipse lies in the direction of 7^h R.A. from us, at a distance of $\frac{7}{16}$ in., or about 90 light years. This estimate may turn out to be the first attempt at a specification of the centre of our system.

Mr. Lewis carried his work still further (though that fact scarcely appears in the volume quoted) by making models of the system in three dimensions, the radii being indicated by wires radiating from a central ball. He was kind enough to give me one of these models, which has been on my table ever since, though I did not realise its possible significance. Measuring the lengths of the wires and analysing the results by spherical harmonics, I find for the direction of the centre from our viewpoint—

$$\text{R.A.} = 113^\circ \quad \delta = +22^\circ,$$

which is not very far from Mr. Eddington's determination of the vertex

$$\text{R.A.} = 94^\circ{\cdot}2 \quad \delta = +11^\circ{\cdot}9.$$

Indeed, considering the absolute independence of the two methods, and the uncertainties of the material in the case of double stars, the accordance is even striking.

Mr. Lewis has thus given us the means of transferring the origin of coordinates to the centre of our system and discussing its ellipsoidal form. But this would take us too far from our

present purpose. We proceed to consider one more piece of independent evidence for the direction of the centre, deduced from actual stellar velocities and their probable changes.

14. It would seem at first sight as though we were completely cut off from the possibility of a *direct* proof of the existence of a gravitational centre, *i.e.* from the detection of actual *acceleration*. The time during which the movements of the stars have been under observation—a couple of centuries at most—is all too short for measuring any change in their movements, as it is for changes in their physical constitution. But, in the latter case, we have attained a considerable measure of success *indirectly*. The existence of stars in various stages of development has enabled us to arrange them in sequence, and thus to picture the probable history of each in long periods of time from what is practically a momentary glimpse. Is any analogous procedure open to us in the case of movement? So long as each star was a mere individual, this would seem indeed hopeless; but recent discoveries of the association of stars in groups seem to open up new possibilities. If we can assume, with any reasonable probability, a common initial velocity for such a group, then perhaps we may detect the effects of acceleration by comparing the velocities of those which have approached nearer the centre with the velocities of those further away?

15. For example, Mr. H. C. Plummer, in a remarkable investigation, which is not yet fully published (but of which a sketch was given in *Monthly Notices*, lxxii. p. 176), has shown that by impressing on the type A stars the condition that their movements shall be parallel to the Milky Way (a condition which is suggested by the observations), the parallaxes and velocities in linear units can be completely deduced; and, moreover, that when this is done, the stars tend to fall into distinct groups. Take, for instance, his group ii. (*loc. cit.*, p. 174). There are 19 stars with approximate velocities

$$U = +2{\cdot}0 \qquad V = +37{\cdot}5 \qquad W = +4{\cdot}8.$$

Of these, six lie far from the two vertices; 8 of the others fall near the 6^h vertex, and the other 5 near the 18^h vertex. The average velocities relatively to the Sun for the vertical groups are—

6^h Vertex . . .	40·3 km. per sec.
18^h Vertex . .	35·8 ,, ,, ,,
Difference . . .	4·5 ,, ,, ,,

16. Is this difference of 4·5 km. real, and if so, how has it been acquired? Is it due to the central attraction acting through a distance of some 500 light years which separates the means of the groups? If so, we may assess the extreme departure of this group which is the swiftest found by Mr. Plummer, at about ten times this interval, say 5000 light years. These figures are at any rate of the same *order* as Seeliger's estimate of 4400 light years for the

distance of the Milky Way: and it is at least worth while to examine Mr. Plummer's results with care. Let us arrange them roughly in order of their remoteness from the centre. Assuming it to lie in the direction

R.A. 94°, Dec. +12°,

and taking Mr. Plummer's axes, x towards 0^h, y towards 6^h, and z to the North Pole, the centre lies near the axis of y. Hence in the first instance we may arrange the stars in the order of y. We shall not exclude any star because of position with reference to the vertex, but we shall omit for the present the four smallest parallaxes, viz.—

	R.A.	Dec.	Parallax.
ν Fornacis . . .	30°	−30°	″·001
L 3081	118	−30	·004
40 Hydræ . . .	150	−13	·004
L 7976	285	−31	·001

which may reasonably be regarded as the weakest results. The remaining 15 stars of his group ii. are as follows, when arranged in order of the y co-ordinate. The velocities U_0, V_0, W_0 are *not* relative to the Sun, but *absolute* so far as we can make them, using $U = -2{\cdot}0$, $V = -12{\cdot}0$, $W = +5{\cdot}6$ for the Sun, as found by Mr. Plummer.

TABLE V.

Co-ordinates and Velocities of Group ii.

Star's Name.	R.A.	Dec.	Par^x.	x.	y.	z.	u_0.	v_0.	w_0.	Total Vel.
	79°	−21°	″·009	+ 20	+101	−40	−3·5	+20·6	+ 1·5	21·0
Lal. 10063	65	+22	·030	+ 13	+ 28	+12	+0·6	+30·9	+13·7	33·8
κ^1 Tauri	65	+17	·036	+ 11	+ 25	+ 8	+0·1	+27·9	+11·5	30·2
64 Tauri	65	+18	·043	+ 9	+ 20	+ 7	+2·0	+24·3	+13·8	28·0
68 Tauri	74	+21	·049	+ 5	+ 18	+ 7	+2·5	+28·1	+16·4	32·6
ι Tauri	40	+10	·040	+ 19	+ 16	+ 4	−2·9	+30·4	+ 6·9	31·3
μ Ceti	153	+43	·022	− 30	+ 15	+31	−1·4	+27·9	+ 8·7	29·3
λ Urs. Maj.	173	−61	·010	− 48	+ 6	−87	−1·4	+17·6	+ 4·3	18·2
Brisb. 3689	189	−48	·025	− 26	− 4	−30	−0·6	+26·4	+ 9·7	28·1
γ Centauri	296	+ 9	·113	+ 4	− 8	+ 2	+2·0	+30·2	+16·2	34·4
α Aquilæ	270	−64	·031	0	− 14	−29	+0·6	+23·7	+10·9	26·1
π Pavonis	318	−54	·020	+ 22	− 20	−40	−1·1	+23·4	+ 7·4	24·6
θ Indi	246	− 8	·043	− 10	− 21	− 3	+1·7	+18·7	+10·8	21·7
ν Ophiuchi	343	+ 8	·009	+108	− 37	+15	+0·2	+28·3	+11·9	30·7
ρ Pegasi	301	− 1	·008	+ 65	−108	− 3	−0·7	+22·8	+ 8·0	24·2
θ Aquilæ										

17. The unit for x, y, z is the distance corresponding to

parallax 1″·0, or about 3 light years. Thus if we take Mr. Lewis' estimate of 90 light years (see § 13) for the *distance* of the centre, or 30 of our units, and take the direction given by Mr. Eddington, the co-ordinates of the centre will be

$$X_0 = -2 \qquad Y_0 = +30 \qquad Z_0 = +7.$$

Subtracting these from x, y, z, and adding the squares of the results, we find the distances of the stars from the adopted centre; and arranging these in order, they compare with the total velocities (given in the last column of Table V.) as follows:—

TABLE VI.

Star's Name.	Dist.	Vel.	Means of Groups. Dist.	Means of Groups. Vel.
θ Aquilæ	−154	24·2		
ρ Pegasi	−130	30·7	130	24·4
Brisbane 3689	−107	18·2		
Lalande 10063	+ 88	21·0		
θ Indi	− 73	24·6	73	23·9
π Pavonis	− 57	26·1		
γ Centauri	− 56	28·1		
υ Ophiuchi	− 53	21·7	47	28·4
α Aquilæ	− 40	34·4		
λ Ursæ Majoris	− 40	29·3		
μ Ceti	− 25	31·3		
κ^1 Tauri	− 16	33·8		
68 Tauri	− 15	28·0	17	31·2
64 Tauri	− 14	30·2		
ι Tauri	− 14	32·6		

[NOTE.—The correction of some small numerical errors makes the grouping now seem singular: but it is still legitimate, and to avoid further corrections it is retained.]

18. The sign attached to the distance is that of the coordinate Y; it will be seen that there is only one star (Lal. 10063) in the list on the far side of the adopted centre. Neglecting this consideration for a moment, there is a clear increase in velocity as the centre is approached.

To attempt to deduce an exact relation between the velocity and the distance is perhaps to treat the material too seriously, but we may thus get at least some notion of the order of the quantities concerned. Our chief difficulty is that the law of attraction is unknown, since we do not know how the mass of the stars is distributed. There is almost certainly an accumulation towards the centre, so that the attraction must vary as some power of the

distance less than the first, and the accumulation is not infinite, so that it is greater than the inverse square.

19. Let us therefore take for trial the four consecutive powers of r + 1, 0, − 1, and − 2.

They give the following formulæ:—

TABLE VII.

(1) Direct distance	$v^2=(29{\cdot}2)^2$	$-r^2\times 0{\cdot}0174$
(2) Constant	$v^2=(30{\cdot}9)^2$	$-r\times 3{\cdot}25$
(3) Inverse distance	$v^2=(41{\cdot}6)^2$	$\log_{10}r\times 577$
(4) Inverse square	$v^2=(23{\cdot}2)^2$	$+7710/r$

and the differences between the velocities calculated according to these formulæ and those "observed" (*i.e.* found above from Mr. Plummer's figures) are as follows:—

TABLE VIII.

Distance.	Observed Velocity.	$C_1-0.$	$C_2-0.$	$C_3-0.$	$C_4-0.$
130	24·4	−1·0	−1·3	−1·8	0·0
73	23·9	+3·5	+2·9	+1·9	+1·5
45	28·4	−0·1	0·0	−0·5	−1·9
17	31·2	−2·3	−1·2	+0·7	+0·3

20. So far as these residuals go, they favour some law very near the inverse square, which suggests a sensible accumulation of mass near the centre. The fourth formula, which gives the best residuals, gives us no indication of the range of motion. But the other three give for the value of r when $v=0$

$$\begin{array}{cccc} & (1) & (2) & (3) \\ r = & 221, & 294, & 300, \end{array}$$

in units corresponding to a parallax of 1″. These are all of the order of about 1000 light years, our adopted unit being about one-third of a light year.

21. We have adopted Mr. Lewis's distance for the centre and Mr. Eddington's direction. Quite possibly some other position could be found which will improve the accordance of the results, though the fact noticed in § 18 that there is only one star on the far side of the centre, and that at some distance, shows that any moderate change of adopted centre will not affect the results very much. And there is this to be said for using values found from independent sources, that at the present preliminary stage of the inquiry it seems desirable to bring together as many converging lines of evidence as possible. But one point is worthy of remark. The most discordant feature of Table VIII. throughout is the

difference between the first two residuals. This is inherent in the velocities for the first two groups, which go the wrong way. Now recurring to Table VI., we see that we could easily make them go the right way by simply transferring the single star Lalande 10063 from the second group into the first.

The mean of the first four stars would then be 23·5, and of the next two 25·3. The grouping is, of course, arbitrary; and the only merit of the actual arrangement is that it is suggested by the distances themselves. It would, no doubt, be better to have no groups at all, and to solve the individual numbers by least squares. This may be undertaken later. But, meanwhile, we may notice that the transference of Lalande 10063 from the second group to the first might follow naturally from a slight change of adopted centre. Suppose, for instance, we put it 25 units away instead of 30. Then for these distant stars the change in radius vector is nearly the same as the change in the Y co-ordinate, so that the distances will now run -149, -125, -102, $+93$, -68, -52, etc., and Lalande 10063 (the fourth in order) is naturally transferred to the first group. Turning back to Table V., we see that the adoption of 25 for the Y co-ordinate of the centre would bring it just into the midst of the Taurus group, instead of just outside it. In any case, the Taurus cluster must be very near the centre of our system, and this fact renders it more intelligible that Professor Boss should have pitched on this particular cluster, with the whole sky to choose from.

22. We must make sure that we are not postulating too great an accumulation of mass at the centre of the system. Whatever may be the value of the material for discriminating between different laws of attraction, it should at any rate give us a fair measure of the attractive force in the neighbourhood of our Sun. Mere inspection of Table VIII. suggests that at distance 30 the acceleration is about 3 km. per second for a diminution of 30 units in distance. Now the unit of distance is 200,000 times the radius of the Earth's orbit, or say 3×10^{13} km.; and thus with a velocity of 30 km. per second 30 units are described in 3×10^{13} secs. Hence the attraction is 10^{-13} km./(sec.)^2. Now the attraction of the Sun at the distance of our Earth is about 6×10^{-6} km./sec.^{2}, and at the distance of 6×10^6 times this amount it would be $1/6 \times 10^{18}$ km./sec.^{2}. Hence the total effective attracting mass on the Sun, made up of all the stars within a sphere radius 30 units round the centre, must be

$$6 \times 10^5 = (85)^3$$

times that of the Sun.

Hence the vacant sphere surrounding each star must have an average radius of about 1/85 of the radius (30 units) of the containing sphere, or about one-third of a unit. The distance from one star to the next will be two-thirds of a unit, corresponding to a parallax of about 1″·5. The crowding is therefore greater, but not much greater than that of which we have experience near the Sun. It

seems reasonable, and tends to confirm the suspicion of an accumulation of mass near the centre.

23. Without presuming to indicate more than the *order* of the quantities concerned, it would appear from the above figure that our Sun was nearest the centre less than 3×10^{13} secs., or a million years ago (perhaps one-half or one-third of this interval). (In this more crowded region did it meet with any vicissitudes?) It is now outward bound again on a journey of some 200 units, say 600 light years, which will occupy it for perhaps 100 million years before it turns towards the centre again. If the geologists are right, we may have passed through the central region more than once since our own planet was formed.

Summary.

§ 1. In a former paper a hypothetical constitution of our stellar system round a centre of attraction was propounded, to explain the "two star streams." On this view the centre should lie in the direction of one of the "vertices." Which?

§§ 2–4. There should be an excess of faint stars near the centre, and a defect at the opposite vertex.

§ 5. Stratonoff's figures (Argelander's) point to the vertex at $94^\circ + 12^\circ$, within reasonable limits.

§ 6. So do counts on the Algiers maps, so far as they go.

§§ 7–10. Basis for more accurate calculations.

§§ 11–13. Mr. Lewis selected this "centre of our universe" by an entirely independent argument from double stars in 1906.

§ 14. Can we trace any acceleration among associated groups of stars, those which are nearer the centre showing larger velocities than those more remote?

§§ 15–21. Mr. H. C. Plummer's figures for his group ii. show just such an acceleration, and give us data for the constants of our system consistent with those of Kapteyn, Eddington, and Lewis.

§ 22. The average "parallax of nearest star" between us and the centre is about $1''{\cdot}5$, taking our Sun as of average mass.

§ 23. The oscillation period of our Sun is about 400 million years, and we passed pericentron less than a million years ago.

The Planet Jupiter in 1889. (Plate 6.)

By A. Stanley Williams.

Full details of the writer's observations of Jupiter in 1887 were published in the year 1889 in a small volume entitled *Zenographical Fragments*, and those made in the following year 1888 in vol. ii. of the same work, published in 1909. The observations of the succeeding opposition of 1889 have now been reduced and discussed in exactly the same manner as those of 1888, and in the present paper is presented a short abstract of the results derived therefrom.

The observations were commenced on 1889 March 2, and terminated on October 12 of the same year. Between these dates 1261 transits of different spots across the central meridian of the planet's disc were recorded. 1168 of these, relating to 93 spots, were actually made use of, the average number of observations to a spot being 12·6. The remaining observations were either duplicate observations, or else could not be certainly identified, or referred to spots observed during so short a period of time as to be valueless for rotation period purposes. The total number of transits is greater than had been obtained in 1888 (viz. 888), notwithstanding that Jupiter's position was even more unfavourable for observation than it had been in that year. The planet was in opposition in 1889 on June 24, in south declination $23\frac{1}{2}^{\circ}$, so that even in the south of England its meridian altitude barely attained to 16°. Moreover, the telescope could not be used for three weeks in May, when it was in course of removal from Hove to Burgess Hill, a place about ten miles inland. The increased number of observations should therefore be ascribed partly to the greater practice or training of the observer—a by no means unimportant factor,—partly to the definition being steadier inland than it was on the actual seaboard, and partly, perhaps, to the fact that the planet was really in a more spotted state than it had been during the preceding year. The observations were all made with the same $6\frac{1}{2}$-in. reflector and a power of 150, excepting a few secured with a $2\frac{3}{4}$-in. refractor during the time when the reflector was dismounted.

No micrometer measures for latitude were obtained in 1889, owing to the very unfavourable position of the planet. But fortunately, the latitudes of the polar edges of the two equatorial belts were measured by the late Prof. G. W. Hough with the $18\frac{1}{2}$-in. Dearborn refractor and the late Prof. J. E. Keeler with the 36-in. refractor of the Lick Observatory. The results of the former observer are taken from an interesting diagram published in *Popular Astronomy*, vol. xi. p. 71. According to this diagram the south edge of the S.E.B. was in lat.*$-21^{\circ}\cdot10$, and the north edge of the

* Computed from the formula, $\sin\phi = d/r$, where d = distance from the centre of the disc, and r = the polar radius of the planet. The correction for the elevation of the Earth above the plane of Jupiter's equator has been applied. Δ is d/r when so corrected.

N.E.B. in + 19°·08. Keeler's measures will be found in a valuable article, "Die Jupiteroberfläche im Jahre 1889," published in *Himmel und Erde,* 1890, ii. 11. These measures make the latitudes − 17°·18 for the south edge of the S.E.B. and + 14°·78 for the north edge of the N.E.B.* The differences between these results are very considerable. A possible explanation of them would be that Hough may have measured the extreme outside irregularities at the edges of the belts, whilst Keeler perhaps measured the average edges of the belts, or even positions still nearer the equator. In other words, different reference points were measured. Whatever may be the reason of the differences, we have here an illustration of the large discordances that may occur between the micrometrical measures of planetary markings made by two independent observers. Taking the means of the above measures, we have—

S. edge of S.E.B. $\Delta = -\cdot 327$, lat. $= -19°\cdot 14$;
N. edge of N.E.B. $\Delta = +\cdot 291$, lat. $= +16°\cdot 93$,

and using these values as basis, the latitudes of the fainter belts and the limits or boundaries of the different surface currents were derived from the drawings.

In order to get some idea of the magnitude of the apparent errors of the observations for longitude, the observations of several of the best observed of the extra-equatorial spots were compared with an ephemeris computed from the adopted rotation periods. The following are the resulting mean errors† of an observed transit:—

Spot.	Mean Error.	No. of Obs.
	m	
Dark northern streak and spot A	±2·8	10
Dark north tropical spot D	±2·9	13
Red spot (middle)	±2·35	11
White south tropical spot α	±3·4	9
Dark south temperate spot β	±1·9	16
Dark south tropical spot ζ	±2·0	15

The average mean error of an observation is ± 2^{m}·6, and having

* Keeler's measures are headed 1888, but it is evident from the context that this must be an error for 1889. The published literature dealing with the planet in 1889 is of the scantiest description, as might only be expected from the unfavourable position of Jupiter in that year, and this article of Keeler's constitutes by far the most important work published. The article is accompanied by eight very fine drawings made with the great Lick refractor. These give an admirable idea of the general appearance of the belts and markings as they were in 1889, and in particular of the complex structure of the principal belts. There are also a few published observations by Denning and others, and the writer has in his possession a few unpublished drawings and notes by different observers, but with these few exceptions there appears to be really nothing.

† As defined by Airy in his work on the *Theory of Errors of Observations,* § 26, p. 19.

regard to the very unfavourable position of the planet in 1889 this cannot be regarded as excessive. But this is, of course, only the apparent error. The best mode of ascertaining how far the results are affected by systematic error of a nature likely to be injurious to the determination of the rotation period, is to compare the observations of the same spot made in two or more consecutive years, so as to see if there is any break in passing from the results of one year to those of another. This, however, necessitates that the spot should be long-enduring, uniform in its motion, and without material change in its appearance, and such spots are unfortunately not very common. Particulars of two instances of the kind have recently been published in the *Observatory*, vol. xxxiv. p. 89, and below will be found the observations of a third spot, a dark projecting spot on the north edge of the N.E.B. as observed in the years 1888 and 1889.*

Dark Spot D. (=Q *of* 1888).

Date.			Transit.	Long.	Wt.	O−C.
			h m	°		m
1888	April	4	15 14	133·7	2	+ 6·7
	May	3	13 54	126·7	2	− 3·9
		10	14 40	127·4	2	− 2·3
		11	10 32	127·9	3	− 1·5
		15	13 49	128·6	2	− 0·1
		20	12 54	127·3	2	− 2·0
		23	10 28	130·3	3	+ 3·2
1889	Mar.	22	16 29	121·0	1	− 0·6
		27	15 36	120·2	2	− 1·6
	June	3	[11 51·7]	[128·1]	dup.	[+14·1]
		27	11 17·5	117·1	1	− 3·2
	July	2	10 30·4	120·6	1	+ 2·8
		19	9 27·4	118·7	2	+ 0·2
		31	9 22·5	119·5	1	+ 1·9

The last column gives the residuals derived from a rotation period of $9^h\ 55^m\ 39^s{\cdot}66$, which pretty well represents the observations of both years, and it will be seen that, within the limits of the errors of the observations, there is no break between the two series of observations. It may therefore be concluded that there is no sensible systematic error of the kind in question. The mean error of an observation from the results of the two years is $\pm 2^m{\cdot}9$, but the observations were unavoidably somewhat rough, and they must not be regarded as an example of the degree of accuracy attainable in eye-estimated transits.

* The longitudes for 1888 have here been increased by 10°, the zero meridian of the ephemeris for 1889 having been shifted by Marth by a corresponding amount.

The Surface Drift of Jupiter in 1889.

The chief aim of the observations, as had been the case in the preceding oppositions of 1887 and 1888, was the determination of the drift of the surface material of Jupiter in different latitudes, by ascertaining the rotation periods or proper motions of the various surface markings of the planet. The results are considered here, as in 1888, under the headings of the different surface currents to which they refer.*

(i.) *The Northern Current.*

In 1888 a number of faint dark streaks had been visible in the northern hemisphere of Jupiter, running in a north and south direction, and radiating to all appearance from the neighbourhood of the North Pole. At the points where these streaks crossed the N.T.B. and two other faint belts farther to the north were generally little darker spots or condensations. A similar series of meridional streaks and spots was observed in 1889. There was perhaps a tendency for these to die out towards the end of the apparition. In some cases the dark spots were seen on the faint belts unaccompanied by the usual meridional streak. The mean rotation period derived from nine of these streaks or spots is $9^h\ 55^m\ 40^s{\cdot}50$. In 1888 nine similar markings gave a period of $9^h\ 55^m\ 39^s{\cdot}43$. Two of the markings seen in 1888 can certainly be identified in 1889. Northwards the meridional streaks certainly extended as far as lat. $+60°$, and southwards they ran beyond the N.T.B., in some cases almost as far as the N.E.B. Lat. $+25°$ has been assumed for the southern limit of the current in 1889, though this is somewhat uncertain, owing to the small difference in velocity of this current and the north tropical current in that year.

(ii.) *The North Tropical Current.*

On the north side of the N.E.B. were many bright and dark spots (see Plate 6, fig. 2). They were usually more or less indefinite and irregular, or at least appeared to be indefinite in the confused seeing generally prevailing. From twelve of these spots (six dark and six bright), the mean rotation period is $9^h\ 55^m\ 39^s{\cdot}25$, as compared with $9^h\ 55^m\ 39^s{\cdot}92$, from fifteen spots in 1888. Unfortunately neither of the bright spots C or L of 1888 can certainly be identified with any of those observed in 1889.† The central rift

* These names were suggested in a very tentative manner in the *Monthly Notices*, vol. lix. p. 33, but as no one else has proposed a better or in fact any other system of nomenclature, the writer may perhaps be excused from using them here, since it is convenient to use something of the kind to distinguish the chief great surface currents of the planet.

† In the *Monthly Notices*, vol. lviii. p. 322, a large number of observations in 1889 and subsequent years were identified with the bright spot C of 1888, but after the present more complete investigation, it is probable that these later observations actually relate to a different spot or spots. At any rate, the identification must be regarded as uncertain at present, though it would still appear to be possible, and the observations of 1890, now in course of reduction, may, it is hoped, definitely decide the question.

of the double N.E.B. (lat. +13°) has been regarded as marking the southern limit of the current in 1889, and the northern limit has been assumed to lie in lat. +25°. But this latter limit is very uncertain, owing to the small difference in velocity of the northern and north tropical currents in 1889.

(iii.) *The Equatorial Current.*

In 1889 there were a great many conspicuous spots, both large and small, and bright and dark, visible in the region included between the two equatorial belts. Plate 6, fig. 2, is intended to give some idea of their general appearance, situation, and arrangement. At times the regular succession of dark and bright north and south equatorial spots had a remarkable regularity and uniformity, so much so that in very confused seeing, and when the markings visible were faint, one was occasionally almost inclined to doubt the evidence of one's own eyes, so artificial and unnatural did this aspect of regularity appear. It may safely be concluded that this unnatural regularity was to a large extent due to the poor seeing usually prevailing at so low an altitude. But making all allowance for this, it is certain that the equatorial markings were more regular and more uniformly placed in 1889 than is the case in most years. The bright spots, it should be mentioned, were generally brightest in the middle, the brighter centre or nucleus being surrounded by fainter radiations. Another point to be noted is that the faint dark equatorial wisps or streaks almost all had a nearly meridional direction, instead of being inclined to the meridian in an angle of roughly about 45°, as had been very generally the case in the two preceding oppositions.

There are several instances of a spot splitting into two spots of the same kind (*i.e.* a white spot splitting or dividing into two white spots), the components gradually separating from one another, and a spot of the opposite kind (*i.e.* a dark spot) making its appearance between the two; and also of two spots of the same kind gradually coming together, the intermediate spot of the opposite kind disappearing or being apparently squeezed out of existence. Similar occurrences have been observed in previous years. Most of the spots, too, exhibited the curious "wanderings" or shifts in longitude which is the peculiar attribute of the equatorial markings, though there is, perhaps, nothing quite comparable with the remarkable shift in longitude of nearly all the equatorial markings which occurred in May 1888.*

The mean rotation period in 1889 from twenty-seven spots on the south side of the N.E.B. is $9^h\ 50^m\ 30^s{\cdot}27$, and that from twenty-seven spots on the north side of the S.E.B. is $9^h\ 50^m\ 30^s{\cdot}19$. Ten of the little dark spots on the narrow discontinuous belt, shown nearly in the middle of the bright equatorial zone, and almost marking the planet's equator, give a rotation period of $9^h\ 50^m\ 36^s{\cdot}02$, but this result is not so reliable as the others, because the spots were only

* See *Zenographical Fragments*, vol. ii. p. 56.

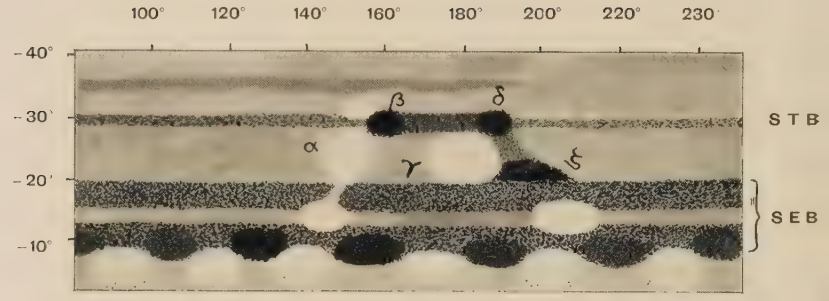


Fig. 1.—South Tropical Disturbance, 1889.

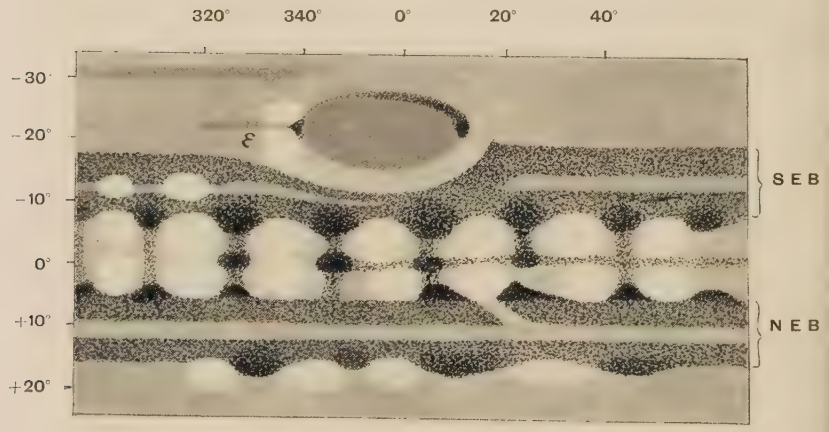


Fig. 2.—Equatorial Regions.

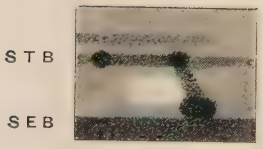


Fig. 3 (1889).

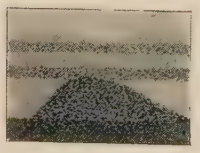


Fig. 4 (1901).
South Tropical Spots.

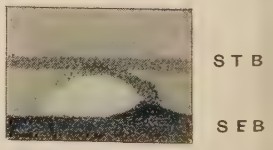


Fig. 5 (1901).

THE PLANET JUPITER, 1889.

observed for a comparatively short period of time, and the motions of the equatorial spots in general were at this time rather slower than the average. The little white spots shown in the central rift of the S.E.B. seem to have moved at about the same rate as the south equatorial spots. The southern boundary of the equatorial current has therefore been taken as coinciding with the south side of this rift (lat. $-15°$), and the northern limit as corresponding to the central rift of the N.E.B. (lat. $+13°$).

(iv.) *The Red Spot Current.*

The red spot itself was by no means a difficult object. It had a distinct pale, pink colour. The sf. margin was edged by a darker border, and there were also little dark spots at both the preceding and following ends (see Plate 6, fig. 2). The following are the observed times of transit of the middle of the spot, with, in the last column, the residuals derived from a rotation period of $9^h\ 55^m\ 40^s{\cdot}13$:—

Date.	Transit.	Wt.	Long.	O−C.
	h m		°	m
1889 Mar. 16	18 03	1	356·2	−0·3
21	17 10	1	355·5	−1·5
26	16 20·5	2	356·9	+0·9
May 30	14 46·5	1	352·1	−5·6
June 4	13 58·5	1	355·2	−0·5
July 3	12 55·3	2	358·6	+5·8
6	10 18·6	2	355·0	−0·2
30	10 04·7	1	354·6	−0·4
Aug. 6	10 54·9	2	356·9	+3·6
11	9 59	2	354·3	−0·5
28	9 02·3	1	353·7	−1·3

The weighted mean time of transit for the date of opposition, 1889 June 24, is $10^h\ 26^m{\cdot}7$, with a probable error of $\pm 0^m{\cdot}59$; the corresponding longitude being $355°{\cdot}22 \pm 0°{\cdot}36$.* The adopted rotation period is slightly shorter than that of 1888 (viz. $9^h\ 55^m\ 40^s{\cdot}58$), but the longitude of the spot computed back to June of that year corresponds to the observed longitude, and there is thus no indication of any break between the two series of observations, suggestive of the existence of systematic error. Transits of the preceding end of the spot were recorded on eleven nights, and of the following end on thirteen nights. On seven nights both ends were observed, and these give $57^m{\cdot}1$ as the time required by the spot to make a complete transit corresponding to $34°{\cdot}5$ of longitude.

* This agrees better with Dr. O. Lohse's curve (*Publications of the Potsdam Astrophysical Observatory*, vol. xxi. Taf. xii.) than the observed value adopted by him from a few observations by Denning and Terby. Lohse himself has no observations of Jupiter in 1889.

Preceding the red spot was the white spot ϵ which had been visible in the preceding year. It was large but ill-defined. The rotation period of this marking was $9^h\ 55^m\ 40^s{\cdot}05$ (ten observations).

(v.) *The South Temperate Current.*

For more than ten years now a remarkable and complex formation or conglomeration of spots and streaks in the southern hemisphere of Jupiter has been familiar to observers of the planet under the name of the *South Tropical Disturbance.* In 1889 a similar formation (here referred to as the South Tropical Disturbance of 1889–92) made its first appearance in the same latitude. In their earlier stages the chief spots of the two disturbances had a marked resemblance to one another. This will be seen from Plate 6, where figs. 1 and 3 show the chief spot of the disturbance of 1889–92, and figs. 4 and 5 two representations of the principal spot of the present existing disturbance. Fig. 4 was drawn by Mr. E. M. Antoniadi with a $9\frac{3}{4}$-in. refractor on 1901 Sept. 30, and fig. 5 by Mr. Scriven Bolton with a $4\frac{1}{8}$-in. refractor on 1901 Oct. 7. But in its later history the disturbance of 1889–92 differed markedly from its successor, and it was moreover comparatively short-lived, and on an inferior scale altogether. Fig. 3 is from a drawing dated 1889, Aug. 5.

The general appearance of this region of the planet in 1889 is represented in Plate 6, fig. 1.* Besides the principal spot ζ there were several other conspicuous bright and dark spots preceding it. Probably all of these should be considered as forming part of the disturbance. It will be seen that the S.T.B. was somewhat broader and darker between the dark spots β and δ than it was elsewhere. This is a very important feature, for it was just here that the very remarkable, short, intensely red streak was visible in 1891, subsequent to the conjunction of the disturbance with the red spot in 1890.

The rotation periods derived from the five chief spots are stated below:—

Spot.	Rot. Per. h m s	Obs.
White spot α	9 55 17·31	9
Dark spot β	9 55 18·20	16
White spot γ	9 55 19·47	17
Dark spot δ	9 55 19·51	17
Dark spot ζ	9 55 16·78	18
Dark spot θ	9 55 18·24	7

* The markings are shown in fig. 1 in the longitudes occupied by them at the date of opposition (1889 June 24). As the daily drift in this latitude amounts to about $\frac{1}{2}°$, there would be a marked displacement in longitude in the course of only a few days. The scale of longitudes in fig. 2 only applies to the region of the Red Spot, and not to the other markings shown. The latter were sketched in without much reference to longitude from various drawings, in order to give some idea of the regular arrangement of the equatorial spots and their peculiar appearance of uniformity in 1889.

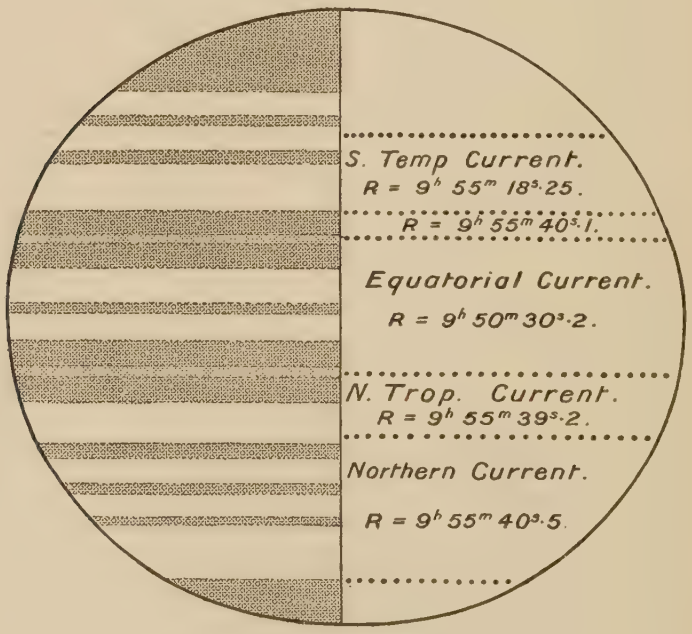
The last spot θ is a little dark spot on the S.T.B. in long. 250° (on June 24) not belonging to the south tropical disturbance. The mean rotation period of the south temperate current in 1889 from the above six spots is $9^h\ 55^m\ 18^s{\cdot}25$. The observations of the chief spot ζ of the disturbance are given below:—

Date.	Transit.	Wt.	Long.	O−C.
	h m		°	m
1889 June 1	[12 43]	1	[218·3]	[+ 9·7]
3	14 09·7	2	211·6	+ 0·5
13	12 12	2	204·5	− 1·5
18	[11 23·7]	1	[207·3]	[+ 7·7]
20	[13 03·6]	2	[208·5]	[+11·6]
25	11 52	3	197·2	− 2·2
27	13 34·5	1	200·0	+ 4·3
30	10 56·1	1	195·4	− 0·4
July 5	9 58·9	2	192·7	− 0·1
17	9 40·1	2	185·6	− 0·3
22	8 39	3	180·4	− 4·2
31	10 57·5	2	176·8	− 1·4
Aug. 15	8 07·3	4	167·8	− 2·0
17	9 45·2	2	167·4	− 0·7
29	9 31·7	3	161·6	+ 1·3
Sept. 1	6 59·4	3	160·1	+ 1·6
8	7 44	2	158·2	+ 5·2
20	7 24·1	2	147·7	− 0·6

The last column contains the residuals derived from the adopted rotation period of $9^h\ 55^m\ 16^s{\cdot}78$. The mean error of an observation is $\pm 2^m{\cdot}0$, omitting the three bracketed observations. It may be noted that the transits were usually observed late when the observing conditions were most unfavourable. The observation of June 20, it is true, is weighted 2, but a note states that Jupiter was partly concealed by a tree at the time, and this very likely may have affected the appearance of the marking. The presence of the oblique streak possibly had something to do with this difference. The south edge of the S.E.B. has been adopted as marking the northern limit of the current in 1889, that is lat. − 19°. Southwards the current certainly extended to the southward of the S.T.B., or to lat. − 32°, but the exact limit is uncertain, since no spots belonging to the southern current were observed in 1889.

Summary for 1889.

Latitude.	Δ.	Current.	Rot. h m s	Per.	Spots.
+60° to +25°	+·87 to +·42	Northern current.	9 55 40·50		9
+25 to +13	+·42 to +·22	North tropical.	9 55 39·25		12
+13 to + 2	+·22 to +·03	Equatorial current.	9 50 30·27		27
+ 2 to − 2	+·03 to −·03		9 50 36·02		10
− 2 to −15	−·03 to −·26		9 50 30·19		27
−19 to −32	−·32 to −·53	South temperate.	9 55 18·25		6
−13 to −28	−·22 to −·47	Red spot.	9 55 40·13		1



Surface Drift of Jupiter, 1889.

The material of the southernmost component of the double S.E.B. probably moves at approximately the same rate as the red spot, but no spots suitable for a definite demonstration of this were observed in 1889. In fact, it is only very seldom that definite markings have been seen in this component of the belt. The above diagram representing the surface drift of Jupiter in 1889 will show at a glance the relationship between the different belts and surface currents of the planet.

1912 *March* 18.

Nova Geminorum 2 (*Enebo*); *its Position for* 1900·0, *together with those for* 102 *Stars surrounding the Nova, as deduced from Measures on a Photograph taken at the University Observatory, Oxford.* By F. A. Bellamy, M.A.

This star was discovered without a telescope by M. Sigurd Enebo, of Dombaas, on March 12, at $8^h\ 32^m$ mean mid-Europe time. Owing to moonlight and cloudy weather no photograph was obtained at this Observatory (with the astrographic telescope of 13 inches aperture) until March 25, when two exposures of $6\frac{1}{2}$ and $6\frac{1}{4}$ minutes were given at $1^h\ 28^m$ west hour-angle; the night was fairly good for photographic work, and a satisfactory plate was obtained.

The plate shows stars to the twelfth magnitude; the images of the *Nova* are very large, being exposed about the period of its second increase in brightness. The object of this note is to record its position, also those of a number of faint stars near the *Nova*, for future use. The area in which all the stars visible on the plate have been measured is situated within about ten minutes of arc radius from the *Nova*; these are mostly stars of the tenth to the twelfth magnitudes. The bright stars on the plate were observed at Leiden; these have been measured and form the basis of the plate-constants: other stars from the eighth to the tenth magnitudes have been measured so as to obtain positions of sufficient stars to afford a good sequence of magnitudes from the seventh to the twelfth, when the *Nova* fades between those limits.

The adopted plate centre is $+32^\circ\ 0'$ and $6^h\ 49^m$ for 1900·0. The rectangular co-ordinates ξ and η have been computed with this centre and referred to the corner of the plate by the addition of 11·0000 *réseau* intervals to the calculated ξ's and η's in order to get ξ' and η'. These standard co-ordinates are given in Table IV., the sixth and seventh columns.

The plate-constants have been determined from 21 stars in the Leiden A.G. Catalogue, reduced to epoch 1900·0, well scattered over the plate. The standard co-ordinates ξ' and η' were calculated by the usual formula by means of MS. tables for 0° to $+64^\circ$ existing at this Observatory, and of similar construction to those already printed in the Astrographic Catalogues (Oxford) for $+25^\circ$ to $+31^\circ$, to which reference may be made for the formulæ used. The residuals for these 21 stars, as well as twelve other Leiden stars not included in the equations from which the plate-constants were determined, are shown in Table I. Notes relating to seven of these stars follow at the end of the Table.

The region under consideration falls on the edges of two plates, Nos. 46 and 47, printed in the *Potsdam Astrographic Catalogue*, vol. iv. The position of the *Nova* would be +8·5 intervals in x from the centre of plate 46; it is outside plate 47.

There are 56 stars, of the 102 mentioned in this paper, that are common to Oxford and Potsdam plates; most of these are the bright stars (Leiden Catalogue) upon which the constants for the plates are based. In the restricted area around the *Nova* I give 45 stars to the twelfth magnitude; 9 of these are on the Potsdam plates. A number of the stars south of $+32^\circ\ 0'$ are also in the *Oxford Astrographic Catalogue*, vol. i., plate 2121.

In Table I. the Potsdam plate and number and B.D. numbers are added for convenience of easy reference.

TABLE I.

Oxford No.	Leiden A.G. Cat.	Potsdam IV.	B.D.	Oxford—Leiden.	
				s	"
1	2841	46, 101	32, 1418	−·09	−0·3
2	2843	46, 107	32, 1419	−·03	−1·6
3	2844	46, 120	32, 1420	−·08	−·6
4	2845	46, 123	32, 1421	·00	+·1
5	2850	...	33, 1421	−·05	−·7
6	2852	46, 157	32, 1424	−·03	−·8
8	2857	46, 185	31, 1428	−·10	−·4
14	2861	46, 215	31, 1433	+·03	−·2
19	2862	46, 221	32, 1433	+·03	+·1
20	2863	...	33, 1428	+·04	+·1
21	2864	46, 223	32, 1434	+·02	+·7
25	2865	46, 227	31, 1434	+·04	−·7
28	2866	46, 229	32, 1436	+·04	+·9
42	2867	46, 243	33, 1429	−·03	+·3
49	2868	46, 245	32, 1437	+·08	+1·0
50	2870	46, 246	31, 1438	−·01	+·1
59	2872	46, 262	31, 1442	+·10	+1·9
61	2873	46, 265	31, 1443	−·08	−0·1
72	2876	46, 268 47, 3	31, 1446	+·12	+0·4
81	2880	46, 275 47, 11	32, 1441	+·04	0·0
82	2883	46, 276 47, 14	32, 1442	−·42	−3·4
83	...	...	32, 1442	−·34	−7·3
87	2887	47, 30	31, 1449	−·08	+·8
89	2889	47, 39	32, 1447	−·05	+2·0
90	2895	47, 54	31, 1453	+·05	+·1
91	2896	47, 55	32, 1448	+·02	+2·1
92	2899	47, 69	33, 1438	+·08	−1·6
93	2901	47, 73	31, 1457	−·27	+1·1

TABLE I.—*continued.*

Oxford No.	Leiden. A.G. Cat.	Potsdam IV.	B.D.	Oxford—Leiden.	
				s	″
94	2902	47, 79	32, 1449	−·02	+1·0
95	2903	47, 80	32, 1450	+·10	−·3
98	2906	47, 107	31, 1460	+·08	−·7
99	2909	...	33, 1446	−·03	+·7
101	2911	47, 128	31, 1463	+·15	+1·1
102	2916	47, 154	31, 1466	+·07	+2·7

Notes.—Nos. 5, 20, and 99 are outside the regions printed in Potsdam astrographic zones in vol. iv.

No. 8. There are three stars within 2ˢ and 50″; in the Potsdam plate 46 they are given as 185 (8·8 mag.), 186 (9·2), and 187 (10·5); only the brightest of the three has been measured at Oxford for this paper.

Nos. 82, 83. These form the double-star Σ 984. Apparently the star was observed as one mass for the Leiden A.G. Catalogue; as there is a footnote, "Keine Bemerkung über Duplicität." Only one other star has been measured at Potsdam on plates 46 and 47, and no note as to duplicity is given. On the Oxford plate 2891 the image of the fainter star coalesces with the brighter image, but it is quite easy to measure; the difference in R.A. is 0ˢ·12 and in Dec. 3″·9, and the photographic magnitudes are 8·5 and 9·9; the companion is *s.f.* It would have been better had this star, Leiden 2883, been omitted from the equations upon which the plate-constants are based; but the whole of the calculations were completed before the magnitude of the residuals was noticed; only the brighter star measured on the Oxford plate has been used in the equations.

No. 93. There is a fainter companion to this star; it is *s.f.*, about 9″ and 11·7 mag.

Only the first exposure was measured. Each star was measured in the direct and reversed position of the plate, and the mean of the measures was taken; these measures were corrected for the plate-constants, and, from these corrected measures (ξ' and η'), the R.A.ˢ and Dec.ˢ for 1900·0 were deduced for the convenience of those who still prefer the results in these co-ordinates. Both sets of results, together with the magnitudes and B.D. number, are given in Table IV.

The formula used for obtaining the magnitudes from the measured diameter (d) in units of ·001 (0″·3) of the *réseau* is $m = a - b\sqrt{d}$; a and b are constants for this plate, and the photographic magnitude scale adopted is

$$\text{photo-mag.} = 13{\cdot}9 - 1{\cdot}02\sqrt{d}.$$

This gives a very fair agreement with Argelander's magnitudes when we assume that his 8·5 = 8·6 and 9·5 = 9·9 as indicated by Professor Pickering in his photometric work. I have only found one star, No. 19 or B.D. + 32° 1433, in the Harvard photometric results that is among the 102 stars used in this paper: that is given as 6·89 mag. It is also the only star given by Müller and Kempf in the Potsdam Observatory's photometric review of B.D. stars; the number is 4243 in vol. xvii., and the star is noted as 7·22 mag. At least two measures of the diameter of each star have been made, and the mean taken to the nearest half unit; these are given in Table IV., second column, and the deduced magnitudes are in the third column.

In addition to the B.D. stars, whose magnitudes and positions are given in Table IV., I have measured the diameters of all the other B.D. stars on the plate, and obtained their magnitudes by the same formula. Collecting those stars corresponding to the 9·5 mag. B.D. stars, the following results are obtained:—

10·4	10·4	9·9	9·6	10·3
10·2	10·4	10·3	10·3	10·5
10·4	9·9	10·2	9·2	9·6
10·1	9·7	10·4	9·5	9·6
9·5	9·6	9·4	9·7	

The mean mag. is 10·0; the groups collected under other magnitudes have insufficient stars. There are only three B.D. stars on the plate with magnitudes brighter than 8·0: they are 7·8 (B.D. 7·8), 7·3 (B.D. 7·8), and 6·9 (B.D. 7·0).

The mean diameter of the *Nova* from ten measures of the first exposure is 80; this, by the same formula, is equal to magnitude 4·8. The semi-diameter, besides the diffusion from the photographic spreading of the image, is 12″; thus all stars within a radius of quite 15″ are lost in the image of the *Nova*.

The following notes upon some of the Argelander stars may be of use:—B.D. + 31° 1425 is given as 9·2 mag.; it is photographically much brighter and, by the size and intensity of the image, is brighter than B.D. + 31° 1433 (7·8 mag.). B.D. + 3°3 1419, given as 9·4 mag., consists of two stars; one is 10·2 and the other *n.f.* * is 10·8. B.D. 31° 1440 is given as 9·2; there are two stars, 9·4 and 10·3 *n.p.* about 33″. B.D. 31° 1445, mag. 9·5, consists of two faint stars, 10·9 and the other 11·0 *n.f.* B.D. 31° 1454, mag. 9·4; there are two stars, mags. 9·5 and 11·6 *n.p.* about 5″. Other notes are appended to Tables I. and III.

For convenience in comparing positions from photographs or from other instruments, the precessions and secular variations for 1900·0 have been computed from Dr. Downing's Tables. Table II. contains this information for a number of points, so

that precessions for intermediate positions can be readily interpolated.

TABLE II.

Precessions and Secular Variations for 1900·0.

In Right Ascension.

	Precession.						Sec. Var. (100 years).	
	+31° 0′	+31° 30′	+32° 0′	+32° 30′	+33° 0′	+33° 30′	+31° 0′	+33° 0′
h m	s	s	s	s	s	s	s	s
6 44	+3·8605	+3·8762	+3·8920	+3·9080	+3·9242	+3·9406	−·0061	−·0068
6 46	·8592	·8748	·8906	·9066	·9227	·9391	64	71
6 48	·8578	·8734	·8891	·9051	·9212	·9375	67	75
6 50	·8563	·8719	·8876	·9035	·9196	·9359	70	78
6 52	·8547	·8703	·8860	·9019	·9179	·9342	73	81
6 54	+3·8531	+3·8686	+3·8843	+3·9002	+3·9162	+3·9324	−·0077	−·0085

In Declination.

	Precession.	Sec. Var. (100 years).		
		+31° 0	+32° 0′	+33° 0′
h m	″	″	″	′
6 44	−3·825	·0·551	−0·555	−0·560
6 46	3·997	·550	·554	·559
6 48	4·168	·548	·553	·557
6 50	4·339	·547	·551	·556
6 52	4·509	·546	·550	·555
6 54	−4·680	−0·544	−0·549	0·553

The position of the *Nova* deduced from the ten bisections of image, is for

	R.A.	ξ′.	Dec.	η
	h m s	R.I.	′ ″	R.I.
1900·0	6 48 25·073	9·5230	+32 15 57·54	14·1928
1912·0	6 49 11·834		+32 15 6·70	

The size of the image, approximately 30″ in diameter, renders the bisections difficult.

In the last two numbers of the *Astronomische Nachrichten*, Nos. 4563 and 4564, some positions of the *Nova* have been published.

It may be of interest to collect these, as some are obtained by means of the meridian instrument, some by micrometer comparisons from one or two bright stars, and from photographs.

	1912·0.	1912·0.	
	h m s	° ′ ″	
Heidelberg	6 49 11·70	+32 15 5·5	Photograph. Measured by F. Kaiser from 2 Leiden stars, 2864 and 2886.
Oxford	6 49 11·83	32 15 6·7	Photograph. Referred to 21 Leiden stars.
Leiden	6 49 11·66	32 15 5·8	Meridian. J. Voûte. Berl. Jahr. and Leiden 2845 used.
Copenhagen	6 49 11·82	32 15 5·5	Meridian. J. Braae.
Rome	6 49 11·82	32 15 6·2	Micrometer. E. Millosevich, from Leiden 2845.
Bamberg	6 49 11·84	32 15 6·0	Micrometer. E. Hartwig, from Leiden 2862.
Hamburg	6 49 11·79	32 15 5·7	Micrometer. K. Graff, from Leiden 2845.
Leiden	6 49 11·70	+32 15 5·1	Micrometer. J. Voûte, from Leiden 2845.

In Table I. I give the corrections to each Leiden star (or the residuals O–L) obtained from this plate. Leiden 2845 (Oxford No. 4) has ^{s}·00 and +″·1; Leiden 2862 (Oxford No. 19) has +^{s}·03 and +″·1; and Leiden 2864 (Oxford No. 21) has +^{s}·02 and +″·7; Leiden 2886 is outside plate 2891.

The two faint stars, 13 and 15 mags., close to the *Nova*, indicated by Dr. Max Wolf as having been measured on one of Dr. Kopff's plates taken in 1909, are lost in the large image of the *Nova* on the Oxford plate.

The magnitude and colour were noted by F. F. Petersen, of Copenhagen, as 5·4 reddish on March 20, 4·7 bluish on March 25, 4·9 on March 26, and 5·8 reddish on March 26. I took plate 2891 on March 25, may this colour effect account for the large image?

Dr. Max Wolf in the *Astronomische Nachrichten*, No. 4562, gives a chart of the region and specially indicates 12 stars; these stars are included in the 102 stars in Table IV., and the following Table III. contains the Oxford number and magnitudes corresponding to the 12 stars in his list, together with the B.D. number and magnitude.

TABLE III.

Oxford.		Max Wolf.	B.D.	
No.	Photo. Mag.	No.	No.	Mag.
10	9·8	1	+32°1431	9·1
11	9·3	2	1432	9·2
19	6·9	3	1433	7·0
21	8·4	4	1434	8·6
23	10·4	5	1435	9·3
28	9·4	6	1436	8·9
49	8·5	7	1437	8·0
73	9·4	8	1439	9·5
80	9·7	9	1440	9·2
81	8·8	10	1441	8·7
82	8·4	11	1442	8·6
85	9·2	12	1443	9·2

Oxford No. 23 is photographically faint, at least it is so on plate 2891.

Table IV. has been explained in the preceding remarks. It contains the results of the measures of 102 stars, besides the *Nova*, tabulated in a convenient form for reference.

TABLE IV.

Oxford.			B.D.		Standard Co-ordinates. 1900·0.		R.A. 1900·0.			Dec. 1900·0.		
No.	Diam.	Mag.	No.	Mag.	ξ'.	η'.	h	m	s	°	′	″
			°		R.I.	R.I.						
1	40	7·4	32, 1418	8·3	−0·908	21·480	6	44	16·47	+32	52	3·8
2	22·5	9·1	32, 1419	9·1	−0·651	11·486	6	44	25·11	32	2	7·3
3	24	8 9	32, 1420	9·1	+0·314	11·636	6	44	47·85	32	2	55·2
4	42·5	7·3	32, 1421	7·8	0·493	14·357	6	44	51·47	32	16	31·9
5	34	7·9	33, 1421	9·0	2·543	25·948	6	45	37·83	33	14	33·4
6	30	8·3	32, 1424	8·8	3·336	22·015	6	45	57·45	32	54	55·9
7	25	8·8	31, 1427	9·2	3·745	4·824	6	46	9·88	31	29	0·2
8	27·5	8·6	31, 1428	8·8	4·928	6·007	6	46	37·47	31	34	57·1
9	30	8·3	31, 1432	9·2	6·747	0·256	6	47	20·68	31	6	14·7
10	16·5	9·8	32, 1431	9·1	6·862	15·125	6	47	22·06	32	20	35·1
11	20	9·3	32, 1432	9·2	6·894	13·983	6	47	22·92	32	14	52·6
12	5·5	11·5	...	...	7·082	15·897	6	47	27·20	32	24	27·0
13	4	11·9	..	...	7·134	13·424	6	47	28·64	32	12	5·2
14	36	7·8	31, 1433	7·8	7·189	1·941	6	47	30·87	31	14	40·5
15	9	10·8	...	...	7·285	14·905	6	47	32·09	32	19	29·6
16	3	12·2	...	...	7·347	14·857	6	47	33·56	32	19	15·2
17	4	11·9	...	...	7·414	14·588	6	47	35·17	32	17	54·6
18	19·5	9·4	...	...	7·424	13·044	6	47	35·52	32	10	11·4
19	47·5	6·9	32, 1433	7·0	7·508	18·617	6	47	37·08	32	38	3·3
20	42·5	7·3	33, 1428	8·3	7·715	24·514	6	47	41·58	33	7	32·2
21	29·5	8·4	32, 1434	8·6	7·721	16·353	6	47	42·30	32	26	44·4
22	13	10·2	...	...	7·892	15·326	6	47	46·42	32	21	36·5
23	12	10·4	32, 1435	9·3	7·919	18·738	6	47	46·84	32	38	40·0
24	7	11·2	...	...	7·932	15·860	6	47	47·34	32	24	16·7
25	23·5	9·0	31, 1434	8·5	7·911	6·531	6	47	47·46	31	37	38·0
26	10	10·7	...	...	8·013	14·843	6	47	49·32	32	19	11·7
27	6	11·4	...	...	8·083	15·713	6	47	50 92	32	23	32·7
28	19·5	9·4	32, 1436	8·9	8·094	17·793	6	47	51·05	32	33	56·7
29	8	11·0	...	...	8·091	14·571	6	47	51·18	32	17	50·1
30	3	12·2	...	...	8·135	12·596	6	47	52·35	32	7	57·7
31	18	9·6	...	..	8·266	15·141	6	47	55·29	32	20	41·2
32	3·5	12·1	...	...	8·396	13·310	6	47	58·47	32	11	32·1

TABLE IV.—*continued.*

Oxford.			B.D.		Standard Co-ordinates. 1900·0.		R.A. 1900·0.			Dec. 1900·0.		
No.	Diam.	Mag.	No.	Mag.	ξ'.	η'.	h	m	s	°	′	″
					R.I.	R.I.						
33	8	11·0	...	...	8·490	13·760	6	48	0·66	32	13	47·1
34	4·5	11·8	...	...	8·523	14·688	6	48	1·40	32	18	25·5
35	3	12·2	...	...	8·563	14·553	6	48	2·35	32	17	45·1
36	5	11·6	...	...	8·824	15·370	6	48	8·49	32	21	50·3
37	19·5	9·4	..	...	8·853	14·093	6	48	9·23	32	15	27·3
38	17	9·7	...	...	8·864	16·392	6	48	9·39	32	26	56·9
39	6·5	11·3	...	...	8·928	14·059	6	48	11·00	32	15	17·1
40	8·5	10·9	..	...	8·929	13·766	6	48	11·04	32	13	49·2
41	8	11·0	...	...	9·042	15·782	6	48	13·63	32	23	54·0
42	23·5	9·0	33, 1429	8·9	9·067	22·793	6	48	13·93	32	58	57·0
43	7	11·2	...	...	9·175	13·832	6	48	16·86	32	14	9·2
44	6·5	11·3	...	...	9·176	12·993	6	48	16·91	32	9	57·5
45	5	11·6	...	...	9·185	13·965	6	48	17·09	32	14	49·1
46	3	12·2	.	...	9·386	12·073	6	48	21·91	32	5	21·5
47	3·5	12·1	...	...	9·387	12·593	6	48	21·91	32	7	57·5
48	6	11·4	...	...	9·437	12·806	6	48	23·08	32	9	1·4
Nova	80	4·8	..	...	9·524	14·152	6	48	25·07	32	15	57·5
49	28	8·5	32, 1437	8·0	9·538	18·331	6	48	25·30	32	36	38·9
50	29	8·4	31, 1438	8·8	9·696	5·266	6	48	29·41	31	31	19·6
51	21	9·2	31, 1437	9·4	9·718	1·854	6	48	30·02	31	14	17·1
52	7	11·2	...	...	9·785	14·150	6	48	31·27	32	15	44·8
53	4·5	11·8	...	...	9·987	14·448	6	48	36·04	32	17	14·3
54	15	9·9	....	...	10·052	12·079	6	48	37·62	32	5	23·6
55	8	11·0	...	...	10·322	13·550	6	48	43·98	32	12	44·9
56	3	12·2	...	...	10·386	14·233	6	48	45·48	32	16	9·8
57	9	10·8	...	...	10·411	13·471	6	48	46·08	32	12	21·2
58	14	10·1	...	...	10·527	12·543	6	48	48·83	32	7	42·9
59	29	8·4	31, 1442	8·6	10·667	9·528	6	48	52·16	31	52	38·4
60	12	10·4	...	...	10·704	12·188	6	48	53·01	32	5	56·4
61	27·5	8·6	31, 1443	8·6	10·921	2·158	6	48	58·15	31	15	47·5
62	3	12·2	...	...	11·217	15·516	6	49	5·14	32	22	34·8
63	19	9·4	...	...	11·221	23·756	6	49	5·27	33	3	46·1
64	5	11·6	...	...	11·296	12·665	6	49	6·99	32	8	19·5
65	13	10·2	...	..	11·475	13·741	6	49	11·23	32	13	42·3
66	4·5	11·8	...	...	11·547	14·954	6	49	12·94	32	19	46·1
67	5	11·6	...	...	11·565	12·690	6	49	13·34	32	8	26·9
68	6	11·4	...	...	11·651	13·728	6	49	15·39	32	13	38·3

TABLE IV.—*continued.*

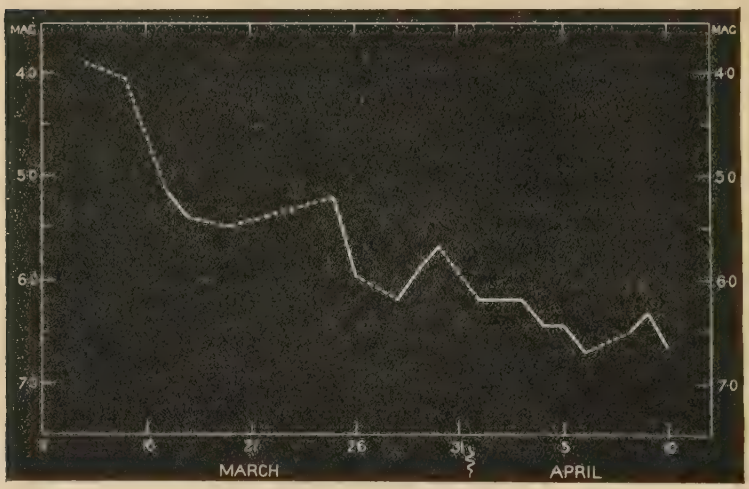
Oxford. No.	Diam.	Mag.	B.D. No.	Mag.	Standard Co-ordinates. 1900·0. ξ'.	η'.	R.A. 1900·0. h	m	s	Dec. 1900·0. °	′	″
			°		R.I.	R.I.						
69	23	9·0	...	...	11·678	21·923	6	49	16·15	32	54	36·6
70	7	11·2	...	...	11·688	14·017	6	49	16·27	32	15	5·0
71	5	11·6	...	...	11·694	12·246	6	49	16·38	32	6	13·7
72	18·5	9·5	31, 1446	8·8	11·764	8·057	6	49	17·97	31	45	17·0
73	19·5	9·4	32, 1439	9·5	11·801	18·187	6	49	19·01	32	35	56·0
74	10	10·7	...	...	11·916	13·959	6	49	21·66	32	14	47·6
75	10·5	10·6	...	...	12·055	12·740	6	49	24·92	32	8	41·9
76	14	10·1	...	...	12·196	17·529	6	49	28·37	32	32	38·4
77	9	10·8	...	...	12·416	17·641	6	49	33·59	32	33	11·9
78	6·5	11·3	...	...	12·439	17·271	6	49	34·13	32	31	20·9
79	6	11·4	...	...	12·448	17·470	6	49	34·34	32	32	20·6
80	17	9·7	32, 1440	9·2	12·588	17·349	6	49	37·66	+32	31	44·3
81	25·5	8·8	32, 1441	8·7	12·727	11·298	6	49	40·73	32	1	29·0
82	29·5	8·4	32, 1442	8·6	13·003	17·917	6	49	47·52	32	34	34·5
83	15	9·9	...	...	13·008	17·904	6	49	47·60	32	34	30·6
84	20	9·3	31, 1448	9·2	13·086	4·153	6	49	48·89	31	25	45·4
85	21·5	9·2	32, 1443	9·2	13·315	18·387	6	49	54·96	32	36	55·3
86	23·5	9·0	33, 1431	9·2	13·379	22·705	6	49	56·70	32	58	30·3
87	30·5	8·3	31, 1449	8·3	14·310	5·182	6	50	17·64	31	30	53·2
88	21	9·2	32, 1445	9·5	14·268	21·711	6	50	17·82	32	53	31·5
89	31·5	8·2	32, 1447	8·3	15·060	16·052	6	50	36·18	32	25	13·3
90	28·5	8·5	31, 1453	8·5	16·588	3·168	6	51	10·84	31	20	46·3
91	18·5	9·5	31, 1455	9·2	16·598	10·788	6	51	11·98	31	58	52·1
92	30·5	8·3	33, 1438	9·0	17·181	23·118	6	51	27·37	33	0	29·6
93	32	8·1	31, 1457	8·5	17·940	5·783	6	51	42·88	31	33	48·4
94	16·5	9·8	32, 1449	9·3	18·191	17·045	6	51	50·50	32	30	6·3
95	24·5	8·9	32, 1450	9·0	18·338	19·631	6	51	54·40	32	43	1·7
96	17·5	9·7	32, 1451	9·5	19·326	12·505	6	52	16·61	32	7	22·0
97	18·5	9·5	31, 1459	9·5	19·835	1·469	6	52	26·55	31	12	10·6
98	24·5	8·9	31, 1460	9·2	20·629	4·396	6	52	45·72	31	26	46·5
99	40	7·4	33, 1446	8·0	21·133	25·375	6	53	2·12	33	11	37·3
100	20	9·3	32, 1453	9·4	21·493	17·114	6	53	8·83	32	30	18·8
101	19	9·4	31, 1463	9·3	22 052	8·876	6	53	20·13	31	49	6·3
102	18·5	9·5	31, 1466	9·4	23·988	1·804	6	54	3·76	31	13	39·1

University Observatory, Oxford:
1912, *April* 10.

Observations of the New Star in Gemini made at the Radcliffe Observatory, Oxford.

(*Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.*)

The Kiel telegram announcing the discovery of this star was received on March 13, and since then observations of its brightness have been made at the Radcliffe Observatory on every evening when the weather permitted. On the 13th, unfortunately, some time was lost in consequence of a mistake in the telegram which stated that the new star was situated in the neighbourhood of η Geminorum, and by the time the error was discovered by Mr. Barrett and the star had been picked up near θ, a short break in



Magnitudes of Nova Geminorum No. 2.

an overcast sky began to close up, and thus an exact determination of brightness became impossible.

From the date of discovery up till March 25 the observations of magnitude were made with the naked eye, and afterwards with the 2·7-inch finder (power 13) of the 10-inch Barclay equatorial. Observations of the colour and character of the image, as described in Mr. Robinson's notes of March 25 and subsequent dates, were made chiefly with the 10-inch refractor (power 85), and on one occasion with the 18-inch refractor (power 290).

In Table I. below are given the reference numbers, the names of the stars used for comparison, and the Harvard magnitudes on which our estimates are based. This is followed by the separate estimations of magnitudes and the observers' remarks; whilst Table II. gives for each evening the mean result obtained by each observer.

Two methods of estimating the brightness have been adopted. The first is Argelander's well-known method of steps; the second, the proportional method, in which the variable is compared with two stars, one brighter and one fainter, and an estimate is made of the ratio which the difference between each comparison star and the variable bears to the difference between the two comparison stars.

The results of the observations are represented graphically in the accompanying figure, which exhibits very clearly variations in the brightness of the star closely resembling but smaller in amplitude than those of Nova Persei (see *Monthly Notices*, vol. lxi.).

The observers were:

Dr. A. A. Rambaut, indicated by A.A.R.
Mr. W. H. Robinson ,, ,, R.
Mr. H. G. S. Barrett ,, ,, H.B.
Mr. J. G. Balk ,, ,, B.

TABLE I.

List of Stars used for Comparison with Nova Geminorum, No. 2.

Ref. No.	Name of Star.	Harvard Photom. Mag.	Ref. No.	Name of Star.	Harvard Photom. Mag.
1	θ Geminorum	3·64	13	28 Geminorum	5·54
2	ι Geminorum	3·89	14	B.D. + 34° 1524	5·60
3	ρ Geminorum	4·18	15	B.D. + 32° 1414	5·76
4	υ Geminorum	4·22	16	48 Geminorum	5·76
5	σ Geminorum	4·26	17	54 Aurigæ	5·84
6	τ Geminorum	4·48	18	53 Geminorum	5·87
7	χ^2 Orionis	4·71	19	B.D. + 29° 1441	5·95
8	64 Geminorum	5·04	20	B.D. + 33° 1433	6·01
9	49 Aurigæ	5·05	21	52 Geminorum	6·02
10	65 (*b*) Geminorum	5·09	22	B.D. + 35° 1511	6·18
11	ω Geminorum	5·21	23	B.D. + 32° 1460	6·46
12	48 Aurigæ (Var.)	5·42	24	B.D. + 32° 1433	6·89

Estimations of Magnitude, and Observers' Remarks

1912. G.M.T.

March 13. $7^h\ 45^m$. No new star was visible near η Geminorum. Suspecting some mistake in telegram, examined neighbourhood of ζ and θ, and found, about 2° south of the latter, a star $\frac{1}{4}$ mag. fainter, which was at first supposed to be a star given in Heis's chart as 6 mag. A more exact estimate was prevented by cloud. (H.B.)

March 15. $7^h\ 15^m$. The object seen on the 13th is undoubtedly the Nova.

March 15. Estimated brightness, in steps of about $0^m{\cdot}05$, as follows:—

$7^h\ 27^m$. θ 4 Nova, Nova 3 v. (H.B.)

$7^h\ 30^m$. Nova $0^m{\cdot}6$ fainter than θ and $0^m{\cdot}5$ brighter than τ. (R.)

$7^h\ 44^m$. θ 4 Nova, Nova 2 v;
ι 4 Nova, Nova 3 ρ. (H.B.)

March 17. $8^h\ 25^m$. Nova has faded considerably since the 15th. It is now fainter than τ by $\frac{1}{2}$ mag. and brighter than No. 15 by $\frac{1}{2}$ mag. (R.)

$10^h\ 30^m$. Nova distinctly fainter than on March 15. It is about $\frac{1}{2}$ mag. below χ^2 Orionis. (B.)

$11^h\ 30^m$. The stars ι, σ, ρ, τ, and 64 Geminorum are all brighter than the Nova, which is very nearly equal to 64 Geminorum. (A.A.R.)

March 18. $11^h\ 30^m$. Sky just cleared, but constellation of Gemini getting low. Nova barely visible. Magnitude about 5·4. (B.)

March 20. $7^h\ 45^m$. Nova brighter than No. 15 by $0^m{\cdot}3$ and fainter than 48 Aurigæ by $0^m{\cdot}3$. (R.) Nova seen only in short breaks of cloud. Brightness about $\frac{1}{6}$ or $\frac{1}{7}$ No. 15 to θ. (H.B.)

$8^h\ 15^m$. Examined Nova with the 10-inch Barclay equatorial, using powers 45 and 85. Image not good; clouds coming up. Nova orange, but no red colour visible. θ Geminorum is white. (R.)

$10^h\ 45^m$. Nova slightly fainter than on March 18. The following observations were made with the naked eye:—

Nova $\frac{1}{4}$ mag. fainter than 64 Geminorum, 65 Geminorum, and 49 Aurigæ; $\frac{1}{2}$ mag. brighter than 53 Geminorum; slightly brighter than 48 and 52 Geminorum; slightly fainter than ω Geminorum; equal to 28 Geminorum, 48 and 54 Aurigæ; $\frac{1}{4}$ mag. brighter than Nos. 14 and 15. (B.)

$11^h\ 0^m$. Nova very faint; magnitude estimated as follows:— $0^m{\cdot}3$ brighter than No. 15; $0^m{\cdot}4$ brighter than No. 19; $0^m{\cdot}2$ fainter than 48 Aurigæ; $0^m{\cdot}3$ fainter than 49 Aurigæ. (R.)

March 25. $7^h\ 0^m$ to $8^h\ 30^m$. Barclay, 10-inch, power 85. The Nova has brightened somewhat since the 20th; its colour, too, has changed, having become redder; the definition was, however, better to-night than on March 20. At $7^h\ 30^m$ the image was noted as orange, with the deep-red fringe characteristic of new stars. The colour of the object contrasts strongly with the white image of θ Geminorum. With the finder Nova estimated about 0·7 or 0·75 θ to No. 15. (R.)

March 25. $7^h\ 47^m$. With finder, Nova = 0·75 or 0·8 θ Geminorum to No. 15.

$7^h\ 55^m$. With the naked eye brightness estimated, as follows:—

τ Geminorum 5 Nova 3 No. 15. (H.B.)

$8^h\ 0^m$. Nova just visible to the naked eye notwithstanding the proximity of the Moon, which, however, was hidden by a shutter of the dome. Could not see No. 15. Nova is $0^m{\cdot}5$ fainter than τ Geminorum. (R.)

March 26. $7^h\ 30^m$. The Nova has diminished in brightness considerably since last evening. In the finder it is $0^m{\cdot}2$ fainter than No. 15. With the 10-inch, power 85, remarked the following:—"The Nova has two foci to-night, one in the red, the other in the pale orange. The focus for the red nucleus with purplish fringe is 4^{mm} longer than that for the yellow, which has a very red and broad margin. Dr. Rambaut also examined the object at $9^h\ 0^m$, and confirmed the above particulars, excepting that he observed a difference of only 3^{mm} between the foci." (R.)

March 28. $7^h\ 30^m$. With finder, No. 15 5 Nova 8 No. 24. The two foci with the 10-inch are distinct, as on the 26th. (R.) Examined the object later in the evening with the 18-inch, power 290. Image yellow with broad and bright-red fringe. Extra focal image has a bright-red nucleus as in 10-inch. (R. and H.B.)

March 29. $7^h\ 30^m$. Nova has brightened about $\frac{1}{3}$ of a magnitude since last evening. In finder, nearly equal to No. 15, say $0^m{\cdot}1$ or $0^m{\cdot}2$ fainter, and about a magnitude brighter than No. 24. Owing to marked difference of colour, the Nova being red and the star No. 15 white or light yellow, they are very difficult to compare for magnitude. In the 10-inch, power 85, Nova's image is yellow with a rich red fringe. Extra-focal image, as before, very red, with a wide purplish border. (R.)

March 30. $7^h\ 15^m$. Nova has brightened slightly since yesterday evening. With finder it is equal to, or possibly rather brighter than, No. 15. Focal and extra-focal images as last night. (R.)

$7^h\ 30^m$. Nova $0^m{\cdot}1$ brighter than No. 15. (A.A.R.)

April 1. $7^h\ 30^m$. Nova has faded again. Brightness, with finder:—

No. 15 5 Nova 8 No. 24.

In 10-inch no change of focal or extra-focal images in colour or character of image. (R.)

$10^h\ 0^m$. With finder, estimated, in steps, as follows:—

No. 15 6 Nova; No. 20 4 Nova; No. 22 3 Nova; Nova 5 No. 23. (H.B.)

April 2. $8^h\ 0^m$. Magnitude same as on last evening. Same stars used for comparison. (R.)

April 3. $7^h\ 30^m$. Same magnitude and colour as on last two evenings. (R.)

April 4. $9^h\ 0^m$. Nova is slightly fainter this evening. Estimations as follows:—

No. 15 6 Nova 4 No. 24.

With the 10-inch, power 85, the extra-focal image is not quite so red as hitherto. At the ordinary focus there is a deep-red fringe as before, but perhaps not so broad. (R.)

April 5. $8^h\ 0^m$. Nova same magnitude as on last evening, using same stars for comparison. Difficult observation in breaks of cloud. Noticed that the Nova was *relatively* fainter by about $0^m{\cdot}1$ or $0^m{\cdot}2$ when seen through cloud than when clear of cloud and compared with the same stars. (R.)

April 6. $7^h\ 30^m$. With the finder made the following estimations:—

No. 15 10 Nova 2 No. 24.

Image in the 10-inch yellow, with deep-red fringe as before; also bright-red extra-focal image. (R.)

April 8. $9^h\ 15^m$. Nova has brightened slightly since the 6th. Estimations with finder:—

No. 15 8 Nova 4 No. 24.

Colour in 10-inch as on April 6. (R.)

April 9. $7^h\ 45^m$. Nova has brightened further. Its magnitude is now midway between those of Nos. 15 and 24. (R.)

April 10. $7^h\ 50^m$. Nova fainter again. With finder, estimated:—

No. 15 10 Nova 2 No. 24. (R.)

$8^h\ 42^m$. With finder:—

No. 23 5 Nova, Nova 4 No. 24. (H.B.)

$10^h\ 50^m$. Nova appears to have brightened since the earlier observation. Estimated as follows:—

No. 23 3 Nova, Nova 6 No. 24;
No. 15 2 Nova 1 No. 24. (H.B.)

TABLE II.

Means of Estimations of Magnitude of Nova Geminorum, No. 2.

1912.	G.M.T. h m	Observer.	Mean Mags.	1912.	G.M.T. h m	Observer.	Mean Mags.
March 13	8 0	H.B.	3·9*	March 28	7 30	R.	6·19
15	7 27	H.B.	3·96*	29	7 30	R.	5·90
15	7 30	R.	4·11*	30	7 15	R.	5·7
15	7 44	H.B.	4·02*	30	7 30	A.A.R.	5·66
17	8 25	R.	5·12*	April 1	7 30	R.	6·19
17	10 30	B.	5·2*	1	10 0	H.B.	6·20
17	11 30	A.A.R.	5·1*	2	8 0	R.	6·19
18	11 30	B.	5·4*	3	7 30	R.	6·19
20	7 45	R.	5·55*	4	9 0	R.	6·44
20	7 45	H.B.	5·44*	5	8 0	R.	6·44
20	10 45	B.	5·47*	6	7 30	R.	6·70
20	11 0	R.	5·48*	8	9 15	R.	6·51
25	7 30	R.	5·18	9	7 45	R.	6·32
25	7 47	H.B.	5·28	10	7 50	R.	6·70
25	7 55	H.B.	5·28*	10	8 42	H.B.	6·70
25	8 0	R.	5·0*	10	10 50	H.B.	6·56
26	7 30	R.	5·96				

* Observed with the naked eye.

Radcliffe Observatory, Oxford:
1912 *April* 11.

The Spectrum of the New Star in Gemini, 1912 *March.*

By W. E. Curtis, A.R.C.S., B.Sc.

(*Communicated by A. Fowler, F.R.S.*)

The first opportunity of investigating the spectrum of the Nova at South Kensington occurred on March 15, when the sky was clear for a short time after dusk. Photographs were taken with the 5-inch equatorial refractor of the Royal College of Science, in conjunction with an objective prism of $12\frac{1}{2}^\circ$ refracting angle.

Two exposures were made, one of 35 minutes duration, from 7.50 to 8.25 p.m., and another of an hour, from 8.35 to 9.35 p.m. The former is rather faint, owing to a too rapid rate of trail, which gave the spectrum a breadth of 2·5 mm. The latter is much more dense, but the exposure was frequently interrupted by clouds. The scale of the spectra is very small (6 mm. from F to K) but a considerable amount of detail can be made out by careful examination.

In its general features the spectrum obviously conformed to

the type which may now be regarded as representative of new stars in the earlier stages of their visibility. That is to say, it comprised broad bright lines, among which those of hydrogen were the most prominent, with very conspicuous dark companions of shorter wave-length. Accurate wave-length determinations were rendered difficult by the small scale of the spectrum and the great breadth of the lines. It should be noted, however, that an increase in dispersion gives still greater breadth to the lines, so that in the case of a spectrum of this nature the effectiveness of the spectrograph, in some respects, is by no means proportional to the dispersion obtained. Both plates were measured independently by Prof. Fowler and myself, and reduced separately, using the bright hydrogen lines as standards. It was noticed that in the case of the most intense bright lines the maximum of intensity did not fall at the centre, but was appreciably displaced towards the more refrangible side; the same peculiarity was found to exist in the spectrum of Nova Aurigæ by Campbell,* and in that of Nova Persei by Lockyer.† On the assumption that the maxima of the fainter lines were similarly displaced, the readings of points one-third the breadth of the bright hydrogen line from the more refrangible edge were adopted in the calculation of the constants in the interpolation formula. In order to test the validity of this proceeding a separate reduction was made, taking the readings of the centres of the hydrogen lines as standards; but the results obtained in this way were distinctly less satisfactory as regards agreement with the wave-lengths of corresponding lines in Nova Aurigæ and Nova Persei. The former method of reduction was therefore adhered to.

The mean values obtained for the wave-lengths of the bright lines are given in the second column of the following table. The third and fourth columns give the wave-lengths of probably corresponding lines in Nova Aurigæ and Nova Persei, as determined by Campbell ‡ and Lockyer § respectively. The probable origins of the lines, as assigned by Lockyer, are given in the fifth column of the table, the prefix *p* indicating that the line is enhanced in passing from the arc to the spark spectrum.

Dark lines which were sufficiently strong and definite to permit of measurement are indicated in the second column of the table by asterisks marking the bright lines with which they appear to have been associated. It is, of course, difficult in some cases to distinguish between lines and interspaces in a spectrum of this character, but the lines indicated are believed to be real.

* *Ast. and Ast. Phys.*, No. 109, Nov. 1892.
† *Roy. Soc. Proc.*, vol. lxviii. p. 234.
‡ *Loc. cit.*
§ *Proc. Roy. Soc.*, vol. lxix. p. 236.

Nova Geminorum 1912, March 15. Description of line.	Wave-length.	Probable corresponding lines. Nova Aurigæ (Campbell).	Nova Persei (Lockyer).	Probable Origin (Lockyer).
Much weaker than H_γ	4861*	4861	4862	$H\beta$
Weak . . .	4823	...	4822	*p* Cr 4824
Moderately strong .	4739	4737	...	
Weak . . .	4634	4630	4628	*p* Fe 4630
Moderately strong .	4587	4586	4583	*p* Fe 4584
Weak . . .	4557	4559 4554	...	*p* Cr 4559 *p* Fe 4556
Weak . . .	4529	4534 to 4502	4535 4519 4509	*p* Ti 4534 *p* Fe 4520 *p* Fe 4508
Moderately strong .	4444	4445	4444	*p* Ti 4444
Strong . . .	4417	4419	4425 4411	*p* Ti 4418 *p* Ti 4411
Weak . . .	4392	4385	4392	*p* Fe 4386
,, . . .	4372	4375	...	*p* Ti 4375
Very strong . .	4341*	4340·6	4341	H_γ
Moderately strong .	4312*	4316	4314	*p* Ti 4315 *p* Sc 4314
Weak . . .	4294*	4296	4300	*p* Fe 4297, 4303 *p* Ti 4300
,, . . .	4270	4267	4262	*p* Cr 4262
Moderately strong .	4249*	4246	4247	*p* Sc 4247
,, ,, .	4229*	4236 4227	4232	*p* Fe 4233
,, ,, .	4210*	4209	...	*p* Sr 4215
Strong . . .	4179*	4180	4175	*p* Fe 4173 4179
Moderately strong .	4126*	4126	4128	*p* Si 4131 4128
Very strong . .	4102*	4102	4102	$H\delta$
Moderately strong .	4079*	4082	...	*p* Sr 4078
Weak . . .	4054*	...	...	
,, . . .	4031*	...	4029	*p* Ti 4029
Strong . . .	3970*	...	3969	$H\epsilon$+Ca(H)
,, . . .	3934*	...	3934	Ca(K)

* These were accompanied by dark lines sufficiently distinct for measurement.

It will be seen that, with the exception of the line 4054, all the bright lines of Nova Geminorum appeared also in Nova Aurigæ or Nova Persei at their corresponding phases. Other lines, not included in the table, were recorded in the previous

new stars, but most of them were weak, and their apparent absence from the photograph under discussion may have been partly due to the small dispersion and under-exposure. The enhanced line of magnesium 4481 is notable among the missing lines.

A general justification for the origins suggested by Lockyer has been obtained by comparing the spectrum of the Nova with that of γ Cygni, in which enhanced metallic lines are a prominent feature. On superposing the negatives so that the hydrogen lines of the comparison star correspond in position with the dark hydrogen lines of the Nova, the coincidences of the dark lines are very striking. This is notably the case with the lines 4031 (*p* Ti), 4079 (*p* Sr), 4128 (*p* Si), 4179 (*p* Fe), and 4312 (*p* Ti, *p* Sc).

As pointed out by several observers in connection with previous new stars, the bright lines are closely allied to those of the solar prominences and chromosphere. The helium lines, however, of which 4472 and 4026 fall in the region investigated, do not appear to have been present in Nova Geminorum on March 15, and the relation of the chromospheric to the nova spectrum is apparently dependent upon the presence of enhanced lines in both.

The average width of the bright hydrogen lines was 16 tenth-metres, and, as remarked in connection with Nova Aurigæ and Nova Persei, the more refrangible edges of the lines were more sharply defined than the edges towards the red. The dark hydrogen lines had an average breadth of 14 tenth-metres, and the mean displacement of the centre of the dark line from the brightest part of its bright companion was about 12 tenth-metres. The accuracy of the measures was not sufficient to indicate the relation between this displacement and the wave-length. With regard to the metallic lines, the average measured distance between the bright and dark components was about 11 tenth-metres, which agrees closely with that determined for the lines of hydrogen when the displacement of the brightest part of the emission line is considered. No certain indication was found that the displacement was different for lines of different elements.

1912, *March* 28.—Unfavourable weather prevented further observations until March 28, and, even then, only occasional glimpses of the Nova were obtained, between 7 and 8 p.m. The star was then very distinctly red in colour, and H_α and H_β were intensely bright. There was a fairly bright line on the more refrangible side of H_β, and many other lines in the green and yellow which were too indistinct for useful estimates of their positions to be made. None of the additional lines were at all comparable with the lines of hydrogen.

1912, *March* 29.—Two exposures were made on this date, when the star was below sixth magnitude. A fairly strong negative was obtained with the first exposure, but the second was very weak on account of clouds. A mere glance at the

spectrum was sufficient to show that considerable changes had taken place since it was photographed a fortnight before. The continuous spectrum was very feeble, and no dark lines were discernible. The bright hydrogen lines β, γ, δ, and ϵ, were very conspicuous, their intensities falling off from β to ϵ in the order named. As H_β was weak relatively to H_γ and H_δ on March 15, it would appear that a shift of the maximum of intensity towards the red had taken place.

Several other bright lines were present in the spectrum, but most of them were too feeble for trustworthy determinations of their positions to be derived. The approximate positions of the brighter lines were 4650, 4615, 4475, 4444, of which the second was the brightest. The first may perhaps be identical with the line at 465 which appears in bright-line stars. The fourth line may be the helium line 4471. At a corresponding stage in Nova Persei, lines were found by Lockyer at approximate wave-lengths 471 (probably helium), 4690, 4650, 4565, and 4472 (probably helium), of which 4650 was described as a very strong broad line.*

In conclusion, I wish to offer my best thanks to Mr. Jevons for assistance in taking the photographs, and to Prof. Fowler for valuable advice and help in discussing them.

Royal Observatory, Greenwich.

Observations of Nova Geminorum No. 2. (Plate 7.)

(*Communicated by the Astronomer Royal.*)

1. From three observations with the transit circle, the position of the Nova for 1912·0 is

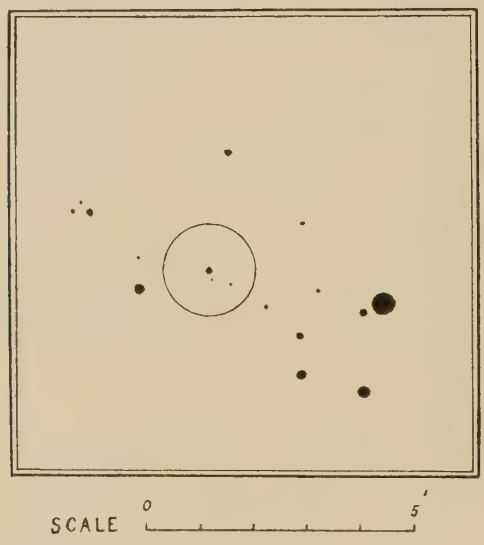
R.A. $6^h\ 49^m\ 11^s{\cdot}85$ Dec. $+\ 32^\circ\ 15'\ 5''{\cdot}6$.

2. Comparison of a photograph taken with the 6-inch Franklin-Adams lens on 1912 March 15 with one taken with the same lens on 1905 February 28, and with one of Mr. Franklin-Adams's chart photographs, confirms the observations made by Dr. Wolf in showing that the Nova may be coincident with one of two small stars of the 15th or 16th magnitude. The identification cannot be made with certainty till the Nova is fainter.

3. The photographic magnitude of the Nova was determined on March 15 by comparison with θ Geminorum, and on other days by comparison with Leiden 2862. With a grating made of strips of zinc 7·5 millimetres wide with intervals of 7·5 millimetres placed in front of the object-glass, photographs of the Nova and Leiden 2862 were taken with the Astrographic telescope. The direct images of the two stars are accompanied by their first

* *Proc. Roy. Soc.*, vol. lxviii. p. 232.

diffraction images which received $\frac{4}{\pi^2}$ of the light of the direct images and are thus $0^m{\cdot}98$ fainter. From the measured diameters of the images, a comparison is made between the magnitude of the Nova and of the comparison star. As an example, eight exposures on plate 8055 taken on March 31 give the following



Map of Field.

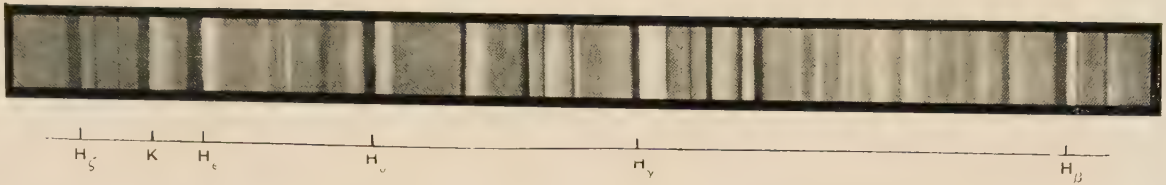
values for the diameters of the Nova (N_0), of Leiden 2862 (L_0), and their first diffraction images (N_1,L_1), the unit being ·005 mm. :—

Exposure.	N_0.	N_1.	L_0.	L_1.
7·5 secs.	22	14	14	6·5
9	23	14·5	16	7
9	23	14	17	8·5
10	25	18	20	11
10	26	16·5	19	11·5
12	28	20	21	10·5
12	27	19	20	11·5
15	27	17	16	6·5
Mean	25·1	16·6	17·9	9·1

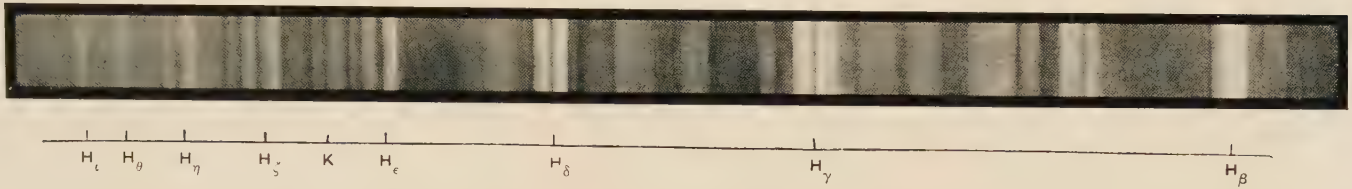
Thus $N_0 - N_1 = 8{\cdot}5$ and $L_0 - L_1 = 8{\cdot}8$.

The adopted difference of diameter for $0^m{\cdot}98$ is 8·75, and thus $L_0 - N_1$, or 1·3, corresponds to a difference of magnitude of $0^m{\cdot}15$.

1912 MARCH 15.



1912 APRIL 3.



SPECTRUM OF NOVA GEMINORUM.—ROYAL OBSERVATORY, GREENWICH.

Therefore the Nova is $0^m{\cdot}98 - 0^m{\cdot}15$, or $0^m{\cdot}83$ brighter than Leiden 2862. The comparison with θ Geminorum on March 15 was made in a similar manner, except that the exposures on the two stars were not simultaneous, but were made first on one star, then on the other. The magnitudes have also been deduced by taking the square roots of the diameters and interpolating in a similar manner. The results by both methods are given below, and the accordance shows the small extent by which the adoption of the formula influences the result.

In the Revised Harvard Photometry the visual magnitude of θ Geminorum is given as 3·64. In the Harvard Photometric Durchmusterung the magnitude of Leiden 2862 is given as $6^m{\cdot}89$. Their spectral types are A2 and A respectively. In the Potsdam Photometry θ Geminorum is given as $3^m{\cdot}84$ W.G., and Leiden 2862 as $7^m{\cdot}22$ G.W.— The photographic magnitudes of the two stars are assumed to be $3^m{\cdot}77$ and $7^m{\cdot}00$ respectively, and on these assumptions the following determinations have been made :—

Plate.	Date.	G.M.T.			Photographic Magnitude of Nova Geminorum.	
					By Linear Interpolation.	By Square Root Formula.
		d	h	m		
8042	March	15	7	30	4·75	4·75
8043	,,	20	7	20	5·67	5·60
8044	,,	21	9	15	6·02	6·02
8046	,,	26	8	20	6·08	6·07
8047	,,	26	9	10	6·13	6·16
8048	,,	26	10	10	6·13	6·17
8049	,,	28	8	10	6·45	6·49
8051	,,	28	11	30	6·37	6·40
8052	,,	29	8	25	6·49	6·59
8053	,,	30	8	25	6·02 *	6·02
8054	,,	31	8	20	6·08	6·09
8055	,,	31	8	40	6·15	6·16
8058	April	1	8	5	6·68	6·68
8061	,,	1	9	0	6·57	6·64
8062	,,	3	8	0	6·27	6·32
8065	,,	3	9	15	6·30	6·34
8068	,,	9	...		6·59	6·56
8074	,,	11	...		6·79	6·78
8078	,,	16	...		6·84	6·80

* This determination is of less weight than others.

4. The spectrum of the Nova was photographed with a 30° flint prism of slant side $4\frac{1}{2}$ inches, and breadth $2\frac{1}{4}$ inches, placed in front of the 6-inch Franklin-Adams lens. The first photograph was obtained on March 15, but owing to cloudy weather no other was

obtained till March 19. Drawings from the photographs obtained on March 15 and April 3 are given. On March 15 the spectrum contained both bright and dark lines. The absorption lines of hydrogen were broad and flanked on the red side by bright lines, and possibly by faint bright lines on the violet side. The calcium line K is very broad. The hydrogen lines are seen as far as H_θ. On March 19 the continuous spectrum was still fairly bright, but the most striking features of the spectrum are the bright hydrogen series, the calcium line at K, and a bright band extending from λ 4670 to 4620. On April 3 a spectrum was obtained which shows clearly the nature of the hydrogen bands, which consist of very strong bright lines displaced to the red, then fine absorption lines, then bright lines on the violet side, and apparently broad absorption lines to the violet. The K line of calcium is shown very faintly. A number of bright bands are visible. By April 11 the continuous spectrum had almost disappeared, and the K line of calcium. The bright band at about λ 4650 is very strong.

The photographs have as yet only been measured in a preliminary manner.

Definition of the Correlation-Coefficient.

By J. C. Kapteyn, Ph.D., Associate R.A.S.

There is no doubt about what is meant by perfect correlation—correlation-coefficient $r = 1{\cdot}00$—between the two varying quantities x and y. It means that any variation of x is accompanied by a definite variation of y. It implies that y is a pure function of x. Neither is there a doubt about what is meant by $r = 0{\cdot}00$, that is, absolute absence of correlation. It means that the variations of x and y are perfectly independent of each other. But what is meant by the statement $r = 0{\cdot}25$? Does it mean a quantity of correlation as that involved in the following simple case?

x and y are both the sum of *four* quantities

$$x = a + b + c + d$$
$$y = a' + b' + c' + d'$$

all varying to exactly the same degree (standard deviations equal). The pair a and a' are perfectly correlated, the other quantities perfectly independent of each other.

It seems certainly plausible to assign to this case a correlation 0·25. But is $r = 0{\cdot}25$?

On those rare occasions that I was led to peruse some articles on correlation I have asked myself the question, but have been unable to find a definition that would settle my doubt. Since, I have been asked the question by others. Finally I requested a

friend, well versed in these matters, to look up the literature on the subject. The result has been negative. We may thus, I think, assume that any acceptable definition does not exist, or at least that it has escaped nearly every worker in this branch of science.

It is true that some authors take the usual formula for the practical computation of the correlation-coefficient:

$$(1) \qquad r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

as a definition.

They try to show that the value on the right-hand side of this equation must be the greater, the greater the quantity of correlation between x and y, and that it can be taken, therefore, as a measure of this correlation. There are, however, two objections to such a course.

First.—If the correlation between two quantities x and y is explained as being the consequence of the fact that x and y are functions partly of common, partly of independent elements—and it is so explained in the theory of Bravais, which seems to have been taken as the base of the study of correlation—then it is natural, and I think logically necessary, to define the correlation-coefficient in terms of such common and independent elements.

Second.—Starting from the simple knowledge that the quantity in the second member of (1) increases and decreases together with the quantity of correlation, we might as well take $\sqrt{r}$ or r^2 . . . . as a measure of correlation. A look, however, at the values of r, r^2, $\sqrt{r}$ in the following table

r.	r^2.	$\sqrt{r}$.	r.	r^2.	$\sqrt{r}$.
0·0	0·00	0·00	0·6	0·36	0·77
0·1	·01	·32	0·7	·49	·84
0·2	·04	·45	0·8	·64	·89
0·3	·09	·55	0·9	·81	·95
0·4	·16	·63	1·0	1·00	1·00
0·5	·25	·71			

will show at once how little satisfactory a definition must be, which will leave us in doubt whether we ought to denote a certain quantity of correlation by such a number as 0·01 or by the number 0·32; or which, in comparing two degrees of correlation, will let us hesitate between

$$\frac{0{\cdot}81}{0{\cdot}01} \quad \text{and} \quad \frac{0{\cdot}95}{0{\cdot}32},$$

that is, between 81 and 3.

Of course, it may be granted that, whichever of the quantities r, r^2, $\sqrt{r}$ . . . . we take, experience will in the long run give us *some* general notion about the question whether such a number as 0·01

denotes a pretty considerable or a very small degree of correlation. But it seems hardly desirable to rely on a vague notion, acquired by long practice, where it must be possible to give a definition in terms of the common elements, which will enable even the "uninitiated" to appreciate what our numbers really mean.

It seems almost unavoidable that the absence of a good definition will keep many an investigator from using r as a measure of correlation. Personally, I know of some cases in which it is so. The consequence must be a serious want of uniformity, which will make the comparison of the results of different authors extremely laborious if not practically impossible. This seems the more to be deplored because, as will be seen further below, the constant r is really a very natural measure of correlation, and can be defined in such a way that its real significance can be grasped at once.

As has already been mentioned, the theory of correlation, as now most generally applied, has been derived in the main from a paper of Bravais, "Analyse mathématique sur les probabilités des erreurs de situation d'un point."* In this beautiful paper Bravais is not really concerned with the quantity of correlation, and for this reason no definition of its coefficient is given. I will simply try, in what follows, to give Bravais' theory the *very small* extension needed for getting at such a definition.

I will consider, with Bravais, only very small deviations; in fact, I will confine myself to the consideration of errors of observation, and will abstain from entering on the question of extending the results to large deviations. In conformity with this plan I will use the terminology generally adopted in the theory of errors of observation.

Let x and y be the coordinates of a point on a plane. The determination of these coordinates by observation will depend on the determination of certain elements as lengths, angles . . . Let $a, b, c, \ldots$ be these directly observed elements. We shall have in general:

$$(2) \qquad \begin{cases} x = \phi(a_1, a_2, \ldots a_\kappa, b_1, b_2, \ldots) \\ y = \psi(a_1, a_2, \ldots a_\kappa, c_1, c_2, \ldots) \end{cases}$$

in which certain of the elements $(a_1, a_2, \ldots a_\kappa)$ are common to both functions, whilst the rest of them $(b_1, b_2, \ldots c_1, c_2 \ldots)$ are different. Let $\alpha_1, \alpha_2, \ldots \alpha_\kappa, \beta_1, \beta_2, \ldots \gamma_1, \gamma_2, \ldots$ represent the errors committed in the observation of $a_1, a_2, \ldots a_\kappa, b_1, b_2, \ldots c_1, c_2, \ldots$

If now we suppose taken as the origin the true position of the point (for which in many cases it may be allowable to take the point for which x and y have their mean value), then the x and y themselves will represent the errors in the situation of our point and we shall have:

* *Mémoires présentés par divers savants à l'Académie des Sciences, Paris*, vol. ix. (1846), pp. 255–332.

$$(3)\qquad \begin{cases} x = A_1\alpha_1 + A_2\alpha_2 + \ldots + A_\kappa\alpha_\kappa + B_1\beta_1 + B_2\beta_2 + \ldots \\ y = H_1\alpha_1 + H_2\alpha_2 + \ldots + H_\kappa\alpha_\kappa + K_1\gamma_1 + K_2\gamma_2 + \ldots \end{cases}$$

in which $A_1 = \dfrac{d\phi}{da_1}$, etc.

We assume that the errors α_1, α_2, . . . of the directly observed quantities fall into Gauss' error curve. It is well known that, this being the case, the x and y will also fall into such curves.

Let ϵ_x, ϵ_y, ϵ_1, ϵ_2, . . . ϵ_κ, ϵ_1', ϵ_2', . . . ϵ_1'', ϵ_2'', . . . represent the mean errors of the quantities

$$x,\ y,\ a_1,\ a_2,\ \ldots\ a_\kappa,\ b_1,\ b_2,\ \ldots\ c_1,\ c_2,\ \ldots$$

then, according to (3),

$$(4)\qquad \begin{cases} \epsilon_x^2 = A_1^2\epsilon_1^2 + A_2^2\epsilon_2^2 + \ldots + A_\kappa^2\epsilon_\kappa^2 + B_1^2\epsilon_1'^2 + B_2^2\epsilon_2' + \ldots \\ \epsilon_y^2 = H_1^2\epsilon_1^2 + H_2^2\epsilon_2^2 + \ldots + H_\kappa^2\epsilon_\kappa^2 + K_1^2\epsilon_1''^2 + K_2^2\epsilon_2''^2 + \ldots \end{cases}$$

If we represent by P_x^{x+dx} the probability that any determination of x will yield a value between x and $x+dx$, then

$$P_x^{x+dx} = \frac{1}{\epsilon_x\sqrt{2\pi}}\, e^{-\frac{x^2}{2\epsilon_x^2}}\, dx$$

and similarly the probability that y will be between y and $y+dy$ by

$$P_y^{y+dy} = \frac{1}{\epsilon_y\sqrt{2\pi}}\, e^{-\frac{y^2}{2\epsilon_y^2}}\, dy\,,$$

in which ϵ_x and ϵ_y are determined by (4).

If the quantities x and y were absolutely independent of each other, that is, if the functions ϕ and ψ had *no* elements α in common, then the probability that x and y will at the same time be contained within the above limits, will simply be the product of the two preceding probabilities, that is, we would have

$$(5)\qquad P_{x,y}^{x+dx,\,y+dy} = \frac{1}{2\pi\epsilon_x\epsilon_y}\, e^{-\frac{x^2}{2\epsilon x^2} - \frac{y^2}{2\epsilon y^2}}\, dx\, dy\,.$$

If, on the contrary, as here supposed, x and y are in part depending on common elements, that is, if they are correlated, this probability will evidently be different. Bravais demonstrates that in this case the probability will be

$$(6)\qquad P_{x,y}^{x+dx,\,y+dy} = \frac{K}{\pi}\, e^{-(ax^2+2exy+by^2)}\, dx\, dy\,,$$

in which the constants a, b, e, K are given by the relations:

$$(7)\quad \begin{cases} a = 2K^2\epsilon_y^2 \\ b = 2K^2\epsilon_x^2 \\ K^2 = ab - e^2 \\ \frac{1}{4K^2} = \epsilon_x^2\epsilon_y^2 - [A_1H_1\epsilon_1^2 + A_2H_2\epsilon_2^2 + \ \ldots\ + A_\kappa H_\kappa \epsilon_\kappa^2]^2 \end{cases}$$

where ϵ_x^2 and ϵ_y^2 stand for the expressions (4). If, for the sake of simplification, we put

$$(8)\qquad r = \frac{1}{\epsilon_x\epsilon_y}[A_1H_1\epsilon_1^2 + A_2H_2\epsilon_2^2 + \ \ldots\ + A_\kappa H_\kappa\epsilon_\kappa^2],$$

consequently

$$(9)\qquad \frac{1}{4K^2} = \epsilon_x^2\epsilon_y^2(1 - r^2),$$

the expression (6) becomes

$$(10)\quad P_{x,\,y}^{x+dx,\,y+dx} = \frac{1}{2\pi}\frac{1}{\sqrt{1-r^2}}e^{-\frac{1}{2(1-r^2)}\left[\frac{x^2}{\epsilon x^2} - 2r\frac{xy}{\epsilon x\epsilon y} + \frac{y^2}{\epsilon y^2}\right]}\frac{dx\,dy}{\epsilon_x\epsilon_y}.$$

Comparing this form with (5), we see that the correlation depends exclusively on the constant r. As, moreover, r becomes *zero* in the case of no correlation at all, and $= 1$ in the case of perfect correlation, it is easily understood how the quantity came to be used as correlation-coefficient.

x and y appear only divided by ϵ_x and ϵ_y.

We thus get a further simplification if we introduce:

$$(11)\quad \begin{cases} \xi = \dfrac{x}{\epsilon_x} \\ \eta = \dfrac{y}{\epsilon_y} \end{cases}$$

or, what comes to the same, if we take

$$(12)\qquad \epsilon_x = 1 \quad \text{and} \quad \epsilon_y = 1,$$

that is, if we express the x and y in their mean amounts as units.*

It may contribute to clearness if we repeat the preceding formulæ on the assumption that, from the beginning, we had expressed the coordinates x and y in the way just found to be

* It seems hardly necessary to point out how natural is this expression in the mean errors as units, especially in cases where the variations of x and y are quantitatively very different. If I am not mistaken, such an expression is very common in all studies on correlation.

expedient, that is, expressed in their mean amounts as units. Suppose therefore,

$$(13) \qquad \begin{cases} \xi = f(a_1, a_2, \ldots a_\kappa, b_1, b_2 \ldots) \\ \eta = \mathrm{F}(a_1, a_2, \ldots a_\kappa, c_1, c_2 \ldots) \end{cases}$$

from which, if we take the true position of the point as our origin :

$$(14) \qquad \begin{cases} \xi = \mathrm{C}_1 a_1 + \mathrm{C}_2 a_2 + \ldots + \mathrm{C}_\kappa a_\kappa + \mathrm{D}_1 \beta_1 + \mathrm{D}_2 \beta_2 + \ldots \\ \quad = \mathrm{T}_1 a_1 + \mathrm{T}_2 a_2 + \ldots + \mathrm{T}_\kappa a_\kappa + \mathrm{L}_1 \gamma_1 + \mathrm{L}_2 \gamma_2 + \ldots \end{cases}$$

where $\mathrm{C}_1 = \dfrac{df}{da_1}$ etc., and further :—

$$(15) \qquad \begin{cases} \epsilon_\xi^2 = 1 = \mathrm{C}_1^2 \epsilon_1^2 + \mathrm{C}_2^2 \epsilon_2^2 + \ldots + \mathrm{C}_\kappa^2 \epsilon_\kappa^2 + \mathrm{D}_1^2 \epsilon_1'^2 + \mathrm{D}_2^2 \epsilon_2'^2 + \ldots, \\ \epsilon_\eta^2 = 1 = \mathrm{T}_1^2 \epsilon_1^2 + \mathrm{T}_2^2 \epsilon_2^2 + \ldots + \mathrm{T}^{\kappa 2} \epsilon_\kappa^2 + \mathrm{L}_1^2 \epsilon_1''^2 + \mathrm{L}_2^2 \epsilon_2''^2 + \ldots \end{cases}$$

We will have :

$$(16) \qquad \mathrm{P}_{\xi,\,\eta}^{\xi+d\xi,\,\eta+d\eta} = \frac{1}{2\pi} \frac{1}{\sqrt{1-r^2}} e^{-\frac{1}{2(1-r^2)}[\xi^2 - 2r\xi\eta + \eta^2]} \delta\xi\, \delta\eta,$$

in which

$$(17) \qquad r = \mathrm{C}_1 \mathrm{T}_1 \epsilon_1^2 + \mathrm{C}_2 \mathrm{T}_2 \epsilon_2^2 + \ldots + \mathrm{C}_\kappa \mathrm{T}_\kappa \epsilon_\kappa^2.$$

Thus far we have done little else than summarise certain of the formulæ of Bravais. The value of r to which it leads must virtually be taken as the definition of the correlation-coefficient. By its means we are already able to answer the question about the value of r in the very simple case considered at the beginning of this note

In the general case, however, the definition yielded by (17) seems too complicated to be quite satisfactory. Happily, it is quite easy to reduce this general case to another in which the simplicity of the definition will leave nothing to be desired. For the formula (16) or (10) proves that, as long as r remains the same and the quantities ϵ_ξ and ϵ_η retain their values, *the whole distribution of the observed positions* over the plane remains the same.

Therefore if ξ and η, instead of being dependent on the elements $a_1, a_2, \ldots b_1, b_2, \ldots c_1, c_2, \ldots$, are made dependent on other elements, satisfying the conditions that :

a. The number of common elements,
b. The quantities $\mathrm{C}_1\mathrm{T}_1\epsilon_1^2$, $\mathrm{C}_2\mathrm{T}_2\epsilon_2^2$, . . ., $\mathrm{C}_\kappa\mathrm{T}_\kappa\epsilon_\kappa^2$ due to the common elements,
c. The total value of ϵ_x and ϵ_y,

all remain the same, then the distribution of the observed points over the plane in the new conditions will be *identically* the same as in the original case.

Such a set of elements may therefore be called an *equivalent set of elements.*

We will make use of this property to substitute for the original set of elements an equivalent set such that the errors ξ and η, instead of being represented by (14), now become:

$$(18)\quad \begin{cases} \xi = \sqrt{C_1T_1}\alpha_1 + \sqrt{C_2T_2}\alpha_2 + \ldots + \sqrt{C_\kappa T_\kappa}\alpha_\kappa + \text{errors due to independent elements} \\ \eta = \sqrt{C_1T_1}\alpha_1 + \sqrt{C_2T_2}\alpha_2 + \ldots + \sqrt{C_\kappa T_\kappa}\alpha_\kappa + \text{errors due to independent elements,} \end{cases}$$

the errors due to the independent elements being such that the total value of ϵ_ξ and ϵ_η are not changed from unity.

We thus have transformed our question to one in which the error in any element common to ξ and η produces the *same* error in both. The definition of r, according to (17), now becomes extremely simple. Before enunciating it, however, I will consider the question from a slightly different point of view.

It is evident that the quantity of correlation depends on the amount of error common to ξ and η as compared to the total quantity of error in these quantities. Now, as long as in (14) the coefficients of the errors of the common elements are the same, there can be no doubt about what is the amount of common error. As soon, however, as the coefficients are different, it is not at once seen what ought to be considered as the common part of the error, what as the independent part. The difficulty is settled by our formulæ. They prove that absolutely nothing will change in the behaviour of the ξ and η relatively to each other if for the two errors $C_i\alpha_i$ and $T_i\alpha_i$ we substitute the common error $\sqrt{C_iT_i}\,\alpha_i$ and at the same time add so much to, or subtract so much from, the independent errors that the total values of ϵ_ξ and ϵ_η remain unchanged.

The quantity $\sqrt{C_iT_i}\alpha_i$, which is the geometrical mean of the errors $C_1\alpha_1$ and $T_1\alpha_1$, must be considered therefore as virtually the common error produced by the error in the common element α_i.

According to (14), the total amount of common error in ξ and η therefore virtually is:

$$\sqrt{C_1T_1}\alpha_1 + \sqrt{C_2T_2}\alpha_2 + \ldots + \sqrt{C_\kappa T_\kappa}\alpha_\kappa.$$

The mean amount of this, viz.,

$$C_1T_1\epsilon_1^2 + C_2T_2\epsilon_2^2 + \ldots + C_\kappa T_\kappa \epsilon_\kappa^2$$

is nothing else than the expression (17) for r.

Remembering that the mean errors of both ξ and η have been taken as units, we thus finally come to this general definition:

The correlation-coefficient r is that fraction of the square of the mean error common to ξ and η which is due to the common elements.

The question remains to be considered whether this square of

the mean error due to the common elements is the natural measure of correlation, or whether it be not, for instance, more expedient to take this mean error itself (that is $\sqrt{r}$) as such a measure.

This question may be settled by a particular example such as that considered at the beginning of this note.

Let

$$\xi = m + n + \ldots + \mu + \nu + \ldots$$
$$\eta = m + n + \ldots + \mu' + \nu' + \ldots$$

and let the elements $m, n, \ldots \mu, \nu, \ldots \mu', \nu', \ldots$ all have the same mean error e. Let a be the number of common elements $m, n, \ldots$ and b the number of independent elements $\mu, \nu, \ldots \mu', \nu', \ldots$, both in ξ and in η, then it seems natural to take

$$\frac{a}{a+b} \tag{19}$$

as the quantity of correlation or the correlation-coefficient.

Our definition assigns to r the value

$$r = ae^2,$$

but we have

$$\epsilon_\xi^2 = \epsilon_\eta^2 = (a+b)e^2 = 1, \text{ therefore } e^2 = \frac{1}{a+b} \text{ and } r = \frac{a}{a+b},$$

which is just the value (19).

The quantity r *itself*, therefore, and not such a function as $\sqrt{r}, r^2, \ldots$ must be considered as the natural measure of correlation.

Recent Observations of Nova Cygni (1876).

By E. E. Barnard.

A measurement of some of the stars near *Nova Cygni* of 1876 which had previously been measured at Dun Echt some twenty odd years before, was undertaken in 1901 with the large telescope, and the results published in *M.N.*, vol. lxii., for March 1902, pp. 405–419. These measures were rather discordant with the earlier ones, and seemed to suggest a possible motion in either the Nova or some of the small stars near it. The meridian circle observations of the star when it was relatively bright in 1877 showed that the motion could not be in the Nova. Sufficient time having elapsed since the measures of 1901 to decide the question of motion, I have remeasured a few of the stars. While the results do not show any certain motion in the small stars, they possibly indicate a very small motion in the Nova itself. One of the bright comparison stars seems to need a correction to the

position as given in the Bonn A.G.C. catalogue, as shown later in this paper. There has been no sensible relative motion of these two stars in the last ten years. The measures of the Nova with the smaller stars, especially with 51 and 60, do not confirm the supposition of motion in the Nova. The difference of 1″ in the measures of its declination in 1901 and 1911 remains unaccounted for, unless both bright comparison stars have moved south by that amount in the interval.

Nova Cygni of 1876.

Nova and 51.

1911·539	July 16	1°·92	66″·71
·552	21	1·61	66·72
·577	30	1·70	66·30
·634	Aug. 20	2·01	66·13
·659	29	1·88	66·48
1911·592		1·82	66·47

Nova and 60.

1911·539	July 16	70°·63	53″·60
·552	21	70·85	53·33
·577	30	70·30	53·10
·634	Aug. 20	70·43	53·12
·659	29	69·86	53·41
1911·592		70·41	53·31

Nova and 47.

1911·539	July 16	205°·70	77″·11
·552	21	205·75	76·98
·577	30	206·21	76·59
·596	Aug. 6	206·33	76·20
·659	29	206·00	76·36
1911·585		206·00	76·65

Nova and 45.

1911·539	July 16	316°·24	105″·64
·552	21	316·19	105·60
·577	30	316·17	105·70
·634	Aug. 20	315·95	105·79
·667	Sept. 1	315·93	105·88
1911·594		316·10	105·72

Nova Cygni of 1876.—continued.

Nova and 41.

1911·539	July 16	293°·98	110″·05
·552	21	293·83	109·88
·577	30	294·13	110·11
·634	Aug. 20	293·88	110·06
·667	Sept. 1	294·37	110·03
1911·594		294·04	110·03

Nova and 54.

1911·552	July 21	175°·49	68″·00	
·582	Aug. 1	175·38	67·81	Very difficult, in haze. Seen only once in a while.
·634	20	175·09	67·78	
·659	29	175·48	67·41	
·667	Sept. 1	175·70	67·61	
1911·619		175·43	67·72	

60 and 68.

1911·582	Aug. 1	57°·50	31″·00
·667	Sept. 1	58·87	31·34
·673	3	57·80	31·25
·826	Oct. 29	57·92	31·29
1911·662		58·02	31·22

Comparing these means, we have:

Nova and 60.

1901·907	70°·34	52″·80	(6 n).
1911·592	70·41	53·31	(5 n).

Nova and 47.

1901·839	206°·19	76″·52	(3 n).
1911·585	206·00	76·65	(5 n).

Nova and 54.

1901·872	175°·70	67″·91	(3 n).
1911·619	175·43	67·72	(5 n).

Nova and 41.

1901·908	294°·15	110″·05	(4 n).
1911·594	294·04	110·03	(5 n).

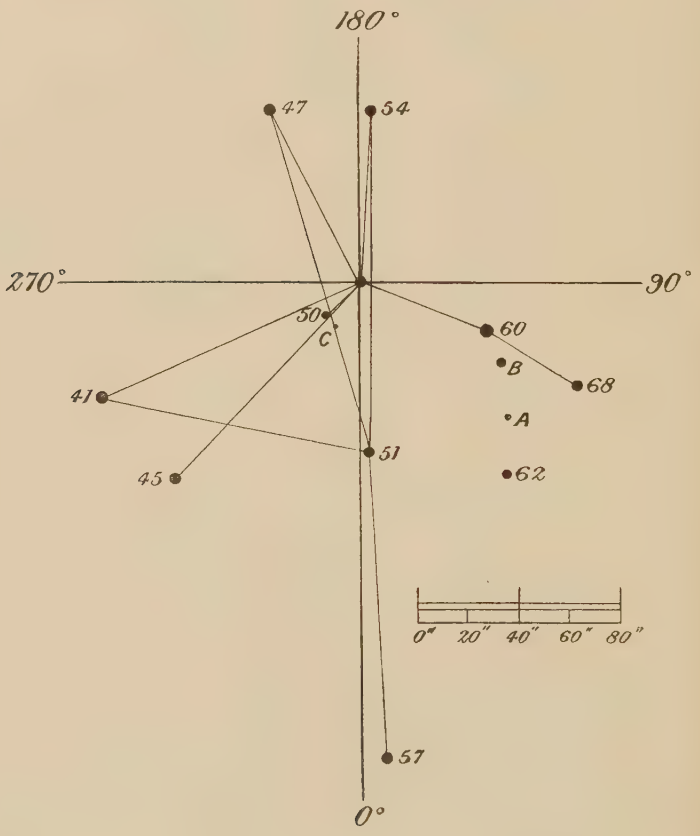
Nova and 45.

1901·987	316°·26	106″·01	(3 n).
1911·594	316·10	105·72	(5 n).

Nova and 51.

1901·875	2°·31	66″·66	(4 n).
1911·592	1·82	66·47	(5 n).

The present observations were made under less favourable conditions than those of 1901, and accordance in the individual results is not so close. Unless the conditions are good the star is difficult to observe. In the observations of 1901 the position of the Nova as derived from the two stars B D $+42^{\circ}$ 4184 and 4185 was discordant by about $0^{s}.2$ in α and $1''$ in δ (*M.N.*, lxii.



p. 413), and it was supposed there might be a small motion in one of these stars. The observations in the present year show that this discordance is due to a want of accuracy in the Bonn A.G.C. position of 4185. The place of 4184 is in accordance with the observations at Greenwich and Dun Echt (*M.N.*, lxii. p. 412).

If the positions of the stars 51 and 60 derived from the stars A and B (*M.N.*, lxii. p. 413) are brought up to the epoch 1911·0 and their differences taken, as below, where A and B indicate the stars on which the position depends, we shall have :—

Δα and Δδ of the Stars 51 *and* 60.

	h	m	s		°	′	″	
A (51)	21	38	13·54		+42	27	14·6	
A (60)			17·77			26	26·0	
	Δα		4·23	Δδ			48·6	Observations of 1901.
B (51)	21	38	13·33		+42	27	13·3	
B (60)			17·59			26	24·8	
	Δα		4·26	Δδ			48·5	

	h	m	s		°	′	″	
A (51)	21	38	13·50		+42	27	15·4	
A (60)			17·77			26	26·8	
	Δα		4·27	Δδ			48·6	Observations of 1911.
B (51)	21	38	13·34		+42	27	14·0	
B (60)			17·56			26	25·1	
	Δα		4·22	Δδ			48·9	

Δα and Δδ of the Stars A and B.

	m	s		′	″	
B (60)	0	30·69		0	57·9	
A (60)		20·31		1	32·3	
	Δα	10·38	Δδ	2	30·2	From the measures of 1911·0. I.
B (51)	0	34·91		0	9·0	
A (51)	0	24·58		2	20·9	
	Δα	10·33	Δδ	2	29·9	

For Δα and Δδ of A and B Observations 1901, *Epoch* 1911·0

	h	m	s		°	′	″	
A	21	38	38·08		+42	24	54·5	
A 51			13·54			27	14·6	
			24·54			2	20·1	(*a*)
A	21	38	38·08		42	24	54·5	
A 60			17·77			26	26·0	
			20·31			1	31·5	(*b*)
B	21	38	48·25		42	27	23·0	
B 51			13·33			27	13·3	
			34·92				9·7	(*c*)
B	21	38	48·25		42	27	23·0	
B 60			17·59			26	24·6	
			30·66				58·4	(*d*)

For $\Delta\alpha$ *and* $\Delta\delta$ *of A and B Observations* 1901, *Epoch* 1911·0—continued.

	$\Delta\alpha$	$\Delta\delta$	
	s	′ ″	
51 − A =	−24·54	+2 20·1	(*a*) and (*c*)
51 − B =	−34·92	−0 9·7	
A − B =	−10·38	−2 29·8	II.
60 − A =	−20·31	+1 31·5	(*b*) and (*d*)
60 − B =	−30·66	−0 58·4	
A − B =	−10·35	−2 29·5	III.

		$\Delta\alpha$	$\Delta\delta$	
		s	′ ″	
Observations of 1901	A − B (II. and III.)	−10·36	−2 29·7	Epoch
,, ,, 1911	A − B (I.)	−10·35	−2 30·0	1911·0

The following observations have been made for the positions of the Nova and the stars 51 and 60. The comparison stars are—

		m		h m s	° ′ ″
A = Bonn A.G.C.	15868	(8·8)	1911·0	21 38 38·08	+42 24 54·5
B = ,, ,,	15876	(9·2)	1911·0	21 38 48·25	+42 27 23·0

Nova A.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 16	...	+1 14·8 (4)
Sept. 1	−0 24·79 (16 transits)	...

Nova B.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 16	...	−1 15·8 (4)
Sept. 1	−0 35·07 (14 transits)	...

51 A.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 16	...	+2 20·9 (4)
Sept. 1	−0 24·58 (10 transits)	...

51 B.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 16	...	−0 9·0 (3)
Sept. 1	−0 34·91 (10 transits)	...

60 A.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 21	...	+1 32·2 (3)
Aug. 1	−0 20·31 (14 transits)	+1 32·3 (3)

60 B.

	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″
1911 July 21	...	−0 57·9 (3)
Aug. 1	−0 30·69 (14 transits)	−0 57·9 (3)

These give the following places:

		h m s	° ′ ″
Nova	1911·0	21 38 13·29	+42 26 9·3 (A)
		13·18	+ 7·2 (B)
51	1911·0	21 38 13·50	+42 27 15·4 (A)
		13·34	+ 14·0 (B)
60	1911·0	21 38 17·77	+42 26 26·8 (A)
		17·56	+ 25·1 (B)

As will be seen, both the right ascension and declination derived from the star A are a little larger than those from B. This was the case with the observations of 1901. The position of A is checked by observations at Greenwich and Dun Echt, and in the declination by earlier observations at Bonn (*M.N.*, lxii. p. 412) there is possibly a small motion in declination of the star B.

No sensible change seems to have taken place in the positions of the stars 51 and 60. This is shown by the deduced $\Delta\alpha$ and $\Delta\delta$ for the epoch 1911·0 as determined from the observations with respect to the stars A and B in the years 1901 and 1911.

	$\Delta\alpha$	$\Delta\delta$
	s	″
From star A (obs. of 1901)	4·23	48·6
,, ,, B ,,	4·26	48·5
	4·24	48·6
From star A (obs. of 1911)	4·27	48·6
,, ,, B ,,	4·22	48·9
	4·24	48·7

In like manner we find for the same epoch (1911·0) the $\Delta\alpha$ and $\Delta\delta$ of the stars A and B through the stars 51 and 60.

From star 51 (obs. of 1901)	10·38	2 29·8
,, ,, 60 ,,	10·35	2 29·7 (3)
	10·36	2 29·8
	s	′ ″
From star 51 (obs. of 1911)	10·33	2 29·9
,, ,, 60 ,,	10·38	2 30·2 (4)
	10·36	2 30·0

The catalogued positions of A and B give :—

	h m s	° ′ ″
A 1911·0	21 38 38·08	+42 24 54·5
B	48·25	27 23·0
Catalogued $\Delta\alpha$ and $\Delta\delta$	10·17	2 28·5
Observed	10·36	2 29·9
Discordance	0·19	1·4

This discordance must be due to error or want of precision in the catalogued position of B. It is not due to proper motion in A or B, as is shown by the accordance of the observations of 1901 and 1911 in (*3*) and (*4*). The position of A is confirmed by the observations at Greenwich and Dun Echt (*M.N.*, lxii. p. 412).

If we assume the position of A to be correct, then the following is the place of B from the observations of 51 and 60 :

B 1911·0 $21^h\ 38^m\ 48^s{\cdot}44$ $+42^\circ\ 27'\ 24''{\cdot}4$

Adopting this position of B, we have the place of the Nova from the present observations :

	h m s	° ′ ″	
With A 1911·0	21 38 13·29	+42 26 9·3	
,, B	13·37	8·6	
Mean	21 38 13·33	+42 26 8·9	(1)

With this new position of B we have for the places of 51 and 60 :

51.

	h m s	° ′ ″
With A 1911·0	21 38 13·50	+42 27 15·4
,, B	13·53	15·4
Mean	21 38 13·52	+42 27 15·4

60.

	h m s	° ′ ″
With A 1911·0	21 38 17·77	+42 26 26·7
,, B	17·75	26·5
Mean	21 38 17·76	+42 26 26·6

From the position angles and distances of the Nova and these stars we get, using the above places,

Position of the Nova.

	h m s	° ′ ″	
With 51 1911·0	21 38 13·33	+42 26 9·0	
,, 60	13·22	8·7	
Mean	21 38 13·27	+42 26 8·8	(2)

With the new position of B the observations of the Nova in 1901 become (*M.N.*, lxii.)

		h m s	° ′ ″	
From 51	1901·0	21 37 49·62	+42 23 24·8	Dependent on B (corrected).
,, 60		49·64	25·1	
,, 51		49·67	24·7	Dependent on A.
,, 60		49·64	24·9	
	1901·0	21 37 49·64	+42 23 24·5	

or

1911·0 $21^h\ 38^m\ 13^s·28$ +42° 26′ 7″·7 From obs. 1901

and from (1) and (2)

1911·0 $21^h\ 38^m\ 13^s·30$ +42° 26′ 8″·8 From obs. 1911.

The discordance of 1″·1 in the declinations of the Nova would seem to show that there has been a slight motion of the star to the north. This, however, must be verified by observations in the future. The measures of the small stars do not verify it.

The position of the Nova derived from the meridian observations in 1877 (*M.N.*, lxii. p. 414) is

1911·0 $21^h\ 38^m\ 13^s·13$ +42° 26′ 7″·1.

The observations upon which this place is based are not strictly accordant (*Copernicus*, vol. ii. p. 115, also *M.N.*, lxii. p. 414). There are discordances as much as $0^s·15$ and 2″·0. It is not safe, therefore, to assume motion from it, but the evidence is in favour of a small motion in the Nova.

The following comparisons of the brightness of the Nova were made during the observations:—

1911 July 16. Very slightly less than 51.
21. 0·2 mag. less than 51. It is closely hazy.
21. 0·3 mag. brighter than 50.
25. 0·2 mag. brighter than 51. It is 14·8 mag. est.
30. 0·25 mag. brighter than 51 It is 14·5 mag. est.
Aug. 20. 0·2 mag. brighter than 51.
20. 0·25 mag. brighter than 50
29. 0·3 less than 51.
Sept. 1. 0·1 brighter than 51.
1. Later, same brightness as 51.
3 0·2 mag. less than 51. Faint, in poor sky.
Oct. 29. Same brightness as 51. Sky thick. Stars faint.

On July 16 a comparison of the focus with No. 51 was made:

Nova.	51.
7·2 mm. (4 comps.)	5·3 mm. (4 comps.)

These show that the focus for the Nova was slightly greater (1·9 mm.) than for 51.

The Nova always appears hazy, like a minute nebula. This ill-defined condition makes it difficult to compare its light with that of the other stars, and moonlight, bad seeing, or haze will make a difference in the estimates. Nevertheless, there is a strong suspicion that it is slightly variable. The same suspicion existed in the observations of 1901 (*M.N.*, lxii. p. 416). Possibly there is a slight variability in No. 51 which will account for the discordances.

The Nova has evidently come to rest, for it does not seem to have faded any in the past ten years. This is evident from the comparisons of its light with 50 and 51. The estimated brightness, 14·8 mag. and 14·5 mag., above, makes it brighter than in 1901; but this does not mean that any change has taken place in its light.

There is a slight error of reduction in the $\Delta\alpha$ and $\Delta\delta$ of 51 and 60 in *M.N.*, lxii. 409.

For $0^m\ 4^s\cdot378$ *read* $0^m\ 4^s\cdot31$
„ $0'\ 47''\cdot78$ „ $0'\ 48''\cdot5$

Yerkes Observatory:
1912 *March* 2.